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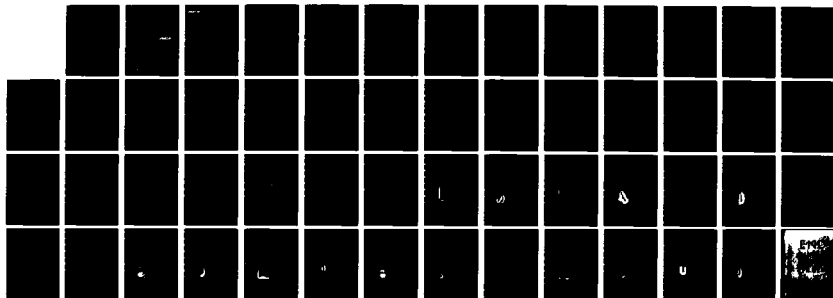
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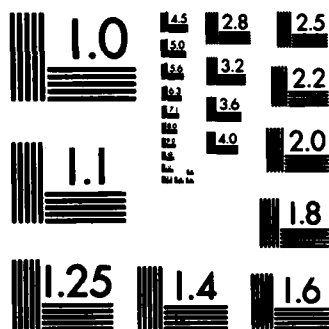
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ELLIPTIC SOLVERS FOR NORDA  
OCEAN MODELS AND A QUASI-GEOSTROPHIC  
REGIONAL EDDY-MEAN ENERGETICS PACKAGE

J206-83-007/6221

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OCEAN MODELS AND A QUASI-GEOSTROPHIC  
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J206-83-007/6221

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by  
Alan J. Wallcraft

August 12, 1983

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## I. ELLIPTIC SOLVER LIBRARY

All two dimensional Helmholtz solvers for rectangles at NORDA have been placed in one library with standardized naming and argument list conventions. Each solver has full internal documentation, test software for accuracy, timing and error conditions (such as incorrect dimensions, etc.). The internal documentation for three representative solvers (SMNUUH, SMNSUH and SMMSSH) is reproduced in Appendix A.

→ To invoke a solver, two routines must be called. The first (e.g., CMNUUH) tests the correctness of the arguments and produces constants for use by the solver proper; it need be called only once for each different Helmholtz operation (i.e., it does not depend on the right hand side of the equation). The second (e.g., SMNUUH) when called with the precalculated constants and a right hand side array actually supplies the solution to Helmholtz's equation, it must obviously be called once for each solution required. These rectangular solvers generally must have the same boundary condition type on the facing parallel sides and uniform grid versions are generally faster than stretched grid solvers so the following convention has been adopted. If the solver is called ABCDEF then:

A - indicates the solver routine type, it can be  
C - for the constant setup routine,  
S - for the acutal solver routine

B - indicates the boundary condition type on the left and right sides; it can be  
P - for periodic,  
D - for Dirichlet,  
N - for (staggered grid) Neumann,  
M - for mixed (Dirichlet and/or Neumann).

C - indicates the boundary condition type on the top and bottom boundaries, it can be P, D, N or M.

D - indicates the grid type in the x-direction; it can be  
U - for uniform grid (constant delta-x),  
S - for stretched grid (variable delta-x).

E - indicates the grid type in the y-direction, it can be U or S.

F - indicates the elliptic operator type; it can be  
H - for Helmholtz's equations,  
S - for separable elliptic equations,  
P - for general positive definite elliptic equations,  
G - for general (non-symmetric) elliptic equations.

The solvers currently in the library are:

- SMNUUH
- SMDUUH
- SMPUUH
- SNMSUH
- SMDSUH
- SMPSUH
- SMMSSH.

By adding a driver routine with appropriate transpose operations, solvers SPMUUH, SNMUSH, SDMUSH and SPMUSH can effectively be created from this set (the appropriate code is contained in the test program).

The uniform-uniform solvers use the FACR(1) algorithm and therefore restrict the active x (or first) dimension to be odd and the y dimension to be of the form 'p' times 2 to the power 'q' plus 'v', where 'p' is usually 3, 4 or 5 (although 7, 9, 11, etc. are possible), 'q' is



a positive integer, and 'v' is -1, 0 or 1 depending on the boundary conditions. They are the fastest solvers in the library and need the least storage for constants. The stretched-uniform solvers use the FACR(0) algorithm, which is very similar to FACR(1) but places no restrictions on the x dimension. They are only slightly slower than the uniform-uniform solvers but require more storage for constants. Stretched-uniform Helmholtz solvers on a rectangle are equivalent to uniform-uniform solvers in the surface of a sphere, and therefore find use in spherical coordinate ocean models. The stretched-stretched solver uses matrix eigenvalue deposition which has no restrictions on the rectangle dimensions but takes a long time to generate the relatively large number of required constants. More importantly, it is an  $O(n^3)$  method rather than the  $O(n^2 \log n)$  performance of the FACR algorithms, and therefore becomes relatively slower for larger problems. On the other hand, the method vectorizes exceptionally well and is very flexible. It is probably the best method available for very small problems (although this may not be very important). Stretched-stretched Helmholtz solvers can be applied to any separable elliptic equation so SMMSSH could equally have been called SMMUUS (or SMMSSS).

Most of the computation within the solvers is performed by two auxillary packages which might be useful in their own right in some applications. The first is a real fast Fourier transform (i.e., fast sine and cosine transform) package by JAYCOR consultant D. R. Moore. This standard FORTRAN code is suitable for automatic vectorization on vector machines and is probably the fastest transportable FFT package available. It contains all the transforms needed by FACR solvers, including the non-standard transform required for staggered grid Neumann boundaries, and is fully documented with its own test routines. The second package contains routines for solving various sets of constant or variable coefficient tridiagonal systems of linear equations. It also is written in standard FORTRAN to allow automatic vectorization and contains full internal documentation.

## II. NON-SEPARABLE ELLIPTIC SOLVERS

The existing stablized marching code was found to be the fastest for the non-separable equations encountered in rigid lid ocean models with bottom topography. But this method has several disadvantages:

- The method is very sensitive to the placement of the 'block' boundaries which artificially divide up the region.
- Storage and time grow with the cube of the dimension of a side of the rectangle. It is therefore best suited to small or mid-size (e.g., 75 x 150) problems.
- Double precision arithmetic must be used throughout. This obviously doubles the storage requirement and in many cases also doubles the CPU time. On the CYBER 205 the highest vectorizable precision is only accurate to about 14 significant figures, so marching methods are at a large disadvantage on this machine.

A diagnostic printout capability has been added to the package to simplify the choice of block boundary positions. The other disadvantages are intrinsic to the method, but note that despite them the method is still the fastest available (at least for mid-size problems).

The best iterative method was found to be 'Black Box' Multigrid [Dendy, 1982]. This method was originally developed for elliptic problems with discontinuous coefficients; the coefficients in ocean model cases are smooth but the discontinuous capability allows non-rectangular problems to be solved by imbedding in a rectangle (with discontinuities at the region boundary). Always solving in rectangles greatly simplifies the task of vectorization, and defining the region via the coefficients removes the necessity for the complicated data structures associated with other non-rectangular multi-grid codes.

However, the cost of this simplification is that a solution is calculated over 'land' where it is not needed (increasing solve time), and additional storage is required because all course grid elliptic operators are 9-point even though the fine grid operator is 5-point.

The total storage required for constants is 7.4 meshes and one 'iteration' takes about as long as one application of the very fast FACR(0) solver (which only works on rectangular Helmholtz problems) and reduces the error by a factor of 10 to 50. Only three or four such iterations are required when used in ocean models so the method is very fast. Marching methods are faster for small problems but the multigrid code has the advantage that both storage and solve time increase only linearly with mesh size. Thus, Black Box multigrid is the method of choice for large problems, and may even be competitive for solving Helmholtz equation in non-rectangular regions (despite the existence of very fast direct Helmholtz solvers).

### III. MODEL COMPARISON

A regional eddy-mean energetics package has been developed for the quasi-geostrophic ocean model. This was tested on a coarse grid (25 x 25) version of a single gyre experiment reported on by Holland [Holland, 1978]. Good agreement was found and sample output from the package is reproduced in Appendix B.

The series of wind driven mesoscale eddy experiments for the model comparison project were completed and the results presented at the 1982 Fall AGU Meeting. The basic finding was that in a classic double gyre experiment the mid-latitude jet develops much more slowly in the quasi-geostrophic results. There were also significant differences in the eddy-mean energetics.

#### REFERENCES

- Dendy, J. E. (1982). "Black Box Multigrid," J. Comp. Phys. 48, pp 366-386.
- Holland, W. R. (1978). "The Role of Mesoscale Eddies in the General Circulation of the Ocean - Experiments Using a Wind-Driven Quasi-Geostrophic Model," J. Phys. Oceanog. 8, pp 363-392.

## APPENDIX A

Sample Internal Documentation from Rectangular Helmholtz Solver Library

- CMNUUH and SMNUUH
- CMNSUH and SMNSUH
- CMMSSH and SMMSSH

```

SUBROUTINE CMNUUH(DAX, DAY, DAC, NDX, NX, NY, Q, LQ, W)
INTEGER NDX, NX, NY, LQ
DOUBLE PRECISION DAX(3,1), DAY(3,1), DAC
DOUBLE PRECISION W
DIMENSION Q(LQ), W(NY,5)

```

```

C
C*****
C*****
C**
C**          INTRODUCTION:
C**
C**          COMPUTES THE CONSTANTS REQUIRED BY SMNUUH TO SOLVE THE
C**          FOLLOWING ( MIXED-NEUMANN ) HELMHOLTZ'S EQUATION ON
C**          AN NX*NY MESH.
C**
C**          FOR I = 1, ..., NX AND J = 1, ..., NY
C**
C**          AN( J ) * H( I, J+1 ) +
C**          AS( J ) * H( I, J-1 ) +
C**          AE( I ) * H( I+1, J ) +
C**          AW( I ) * H( I-1, J ) +
C**          AP( I, J ) * H( I, J ) = S( I, J )
C**
C**          WHERE
C**          AN( J ) = DAY(3,1) J=1...NY-1,
C**                   0 J= NY.
C**          AS( J ) = 0 J= 1.
C**                   DAY(3,1) J=2...NY,
C**          AE( I ) = DAX(3,1) I=1...NX-1,
C**                   0 I= NX.
C**          AW( I ) = 0 I= 1.
C**                   DAX(3,1) I=2...NX,
C**          AP( I, J ) = DAC+DAX(2,1)-(AN(J)+AS(J)+BE(I)+BW(I))
C**                                     I=1...NX, J=1...NY.
C**          BE( I ) = DAX(3,1) I=1...NX-1,
C**                   DAX(1,1) I= NX.
C**          BW( I ) = DAX(1,1) I= 1.
C**                   DAX(3,1) I=2...NX,
C**
C**          USUALLY DAX(2,1)=0.DO .
C**
C**          FOR DIRICHLET-NEUMANN PROBLEMS DAX(1,1)=DAX(3,1) ,
C**          FOR NEUMANN-NEUMANN PROBLEMS DAX(1,1)=0.DO .
C**
C**          FOR COMPATABILITY WITH OTHER SOLVERS SET DAY(1,1)=0.DO
C**                                     AND DAY(2,1)=0.DO .
C**
C*****
C**
C**          ARGUMENT LIST: SEE OTHER COMMENTS FOR DETAILS
C**
C**          DAX, DAY, DAC - HELMHOLTZ COEFFICIENTS
C**
C**          NDX - 1ST ARRAY DIMENSION OF S AND H IN THE SOLVER
C**
C**          NX, NY - DIMENSIONS OF MESH (NX ODD AND NY=PA2**L)
C**
C**          Q(LQ) - ARRAY FOR REQUIRED CONSTANTS
C**
C**          LQ - SIZE OF Q, MUST BE AT LEAST LQ1+LQ2+5, WHERE:
C**                  LQ1 = NY + 10*LQ3**2 + 1
C**                  LQ2 = NY*(NX+7)/4 + (NY+3)/2 + 2
C**                  LQ3 = 0 IF P = 4, 5, OR 6;

```

```

C**              (P-1)/2  IF P = 7 OR GREATER              **
C**                                                    **
C**  W(NY,5)      -  DOUBLE PRECISION WORKSPACE ARRAY      **
C**                                                    **
C*****                                                    **
C**              SPECIAL FEATURES OR NON-ANSI USAGE          **
C**                                                    **
C**  AN ERROR MESSAGE WILL BE OUTPUT TO CHANNEL 6, AND THE PROGRAM **
C**  TERMINATED, IF ANY OF THE FOLLOWING CONDITIONS ARE DETECTED: **
C**      NX              LESS THAN 5 OR EVEN                 **
C**      NY              INCOMPATABLE WITH THE FOURIER PACKAGE **
C**      LQ              TOO SMALL                           **
C**      DAX,DAY,DAC     NOT CONSISTANT WITH HELMHOLTZ'S EQUATION **
C**                                                    **
C**  THE WORKSPACE ARRAY, W, NEED NOT BE DISTINCT FROM THE   **
C**  WORKSPACE SUPPLIED TO THE SOLVER.  HOWEVER THE LATTER MAY **
C**  CONSIST OF SINGLE PRECISION ARRAYS.                     **
C**                                                    **
C**  FOR MORE DETAILED DOCUMENTATION -  SEE SMNUUH.          **
C**                                                    **
C*****                                                    **
C*****                                                    **
C
      .
      .
      .
C  END OF CMNUUH.
END

```



```

SUBROUTINE SMNUUH(S,H,NDX,NX,NY, Q,LQ, W1,W2)
INTEGER NDX,NX,NY, LQ
DIMENSION S(NDX,NY),H(NDX,NY)
DIMENSION Q(LQ)
DIMENSION W1(1), W2(1)
C.....DIMENSION W1(NX+1,NY),W2(NX,NY)
C
C*****
C*****
C**
C**      DATE:      SEPTEMBER 1978
C**
C**      AUTHOR:    D.R. MOORE,  A.J. WALLCRAFT
C**
C**      ARCHIVIST: A.J. WALLCRAFT
C**
C**      ADDRESS:   NORDA
C**                  CODE 322
C**                  NSTL STATION, MS 39529
C**
C**      PHONE:    601-688-4835  (COMMERCIAL)
C**                  494-4835    (FTS)
C**                  485-4835    (AUTOVON)
C**
C**      DATE OF CURRENT SEQUENCE:  DECEMBER 1982
C**
C*****
C**
C**      INTRODUCTION:
C**
C**      SOLVES FINITE DIFFERENCE HELMHOLTZ'S EQUATIONS:
C**      OVER A RECTANGLE IN CARTESIAN COORDINATES,
C**      ON A  UNIFORM-UNIFORM  MESH,
C**      WITH (ZERO) NEUMANN BOUNDARY CONDITIONS 1/2 GRID DISTANCE
C**      OUTSIDE THE MESH AT J=1 AND J=NY,
C**      AND EITHER (ZERO) NEUMANN BOUNDARY CONDITIONS 1/2 GRID
C**      DISTANCE OUTSIDE THE MESH OR (ZERO) DIRICHLET BOUNDARY
C**      CONDITIONS 1 GRID DISTANCE OUTSIDE THE MESH AT I=1 AND I=NX.
C**
C**      USES THE (UNSTABILISED) FACR(1) METHOD.
C**
C**      BOTH THE INPUT ARRAY S(NDX,NY) AND THE OUTPUT ARRAY H(NDX,NY)
C**      ARE ASSUMED TO CONTAIN ACTIVE SUBARRAYS FROM 1 TO NX AND
C**      1 TO NY.  THE NON-ACTIVE SUBARRAY, NX+1 TO NDX AND 1 TO NY,
C**      OF S CAN BE ARBITRARY ON INPUT AND THAT OF H IS  RETURNED
C**      INTACT.
C**
C**      IN THE CALLING SEQUENCE THE INPUT AND OUTPUT ARRAYS MAY BE THE
C**      SAME ARRAY.  IF THEY ARE NOT THE SAME, THE INPUT ARRAY IS
C**      RETURNED INTACT BY THE SOLVER.
C**
C**      IT IS ASSUMED THAT NECESSARY CONSTANTS HAVE BEEN PRECOMPUTED
C**      EXTERNALLY AND ARE STORED IN THE ARRAY Q(LQ), WHICH
C**      IMPLICITLY DEFINES THE HELMHOLTZ COEFFICIENTS (DAX, DAY, DAC)
C**      TO THE SOLVER.
C**
C**      FOR I = 1, ..., NX  AND  J = 1, ..., NY  THE SOLUTION WILL
C**      SATISFY:
C**
C**      AN( J ) * H(I, J+1) +
C**      AS( J ) * H(I, J-1) +
C**      AE(I ) * H(I+1,J ) +

```

```

C**          AW(I ) * H(I-1,J ) +
C**          AP(I,J) * H(I, J ) = S(I,J)
C**
C** WHERE
C**      AN(J)  = DAY(3,1)  J=1...NY-1,
C**              0          J= NY.
C**      AS(J)  = 0          J= 1.
C**              DAY(3,1)  J=2...NY,
C**      AE(I)  = DAX(3,1)  I=1...NX-1,
C**              0          I= NX.
C**      AW(I)  = 0          I= 1.
C**              DAX(3,1)  I=2...NX,
C**      AP(I,J) = DAC+DAX(2,1)-(AN(J)+AS(J)+BE(I)+BW(I))
C**                                  I=1...NX, J=1...NY.
C**      BE(I)  = DAX(3,1)  I=1...NX-1,
C**              DAX(1,1)  I= NX.
C**      BW(I)  = DAX(1,1)  I= 1.
C**              DAX(3,1)  I=2...NX,
C**
C*****
C** LIBRARIES REQUIRED:
C**
C** FAST FOURIER TRANSFORM LIBRARY BY D.R. MOORE
C** TRIDIAGONAL LINEAR EQUATION SOLVER LIBRARY BY A.J. WALLCRAFT
C**
C*****
C** ARGUMENT LIST: SEE OTHER COMMENTS FOR DETAILS
C**
C** S(NDX,NY) - INPUT ARRAY
C**
C** H(NDX,NY) - OUTPUT ARRAY
C**
C** NDX,NY - SIZES OF S AND H
C**
C** NX - ACTIVE SUBDIMENSION OF S AND H
C**
C** Q(LQ) - CONSTANT ARRAY, PRECOMPUTED EXTERNALLY
C**
C** LQ - DIMENSION OF Q
C**
C** W1(NX+1,NY) - WORKSPACE ARRAYS,
C** W2(NX, NY) RETURNED AND AVAILABLE AFTER EACH CALL.
C**
C*****
C** SPECIAL FEATURES OR NON-ANSI USAGE:
C**
C** DOUBLE PRECISION LIBRARIES USE THE IMPLICIT STATEMENT
C**
C** TI-ASC ONLY: USE W-OPTION WHEN COMPILING FOURIER PACKAGES
C**
C** NX MUST BE ODD AND .GE. 5,
C** NY MUST BE OF THE FORM P*2**L, WITH L = 1, ..., 7.
C** USUALLY P=4, 5, OR 6 -
C** HIGHER VALUES OF P ARE POSSIBLE, BUT ARE LESS EFFICIENT.
C** LQ MUST BE AT LEAST LQ1+LQ2+5 (SEE CMNUUH).
C**
C** PROBLEMS WITH NON-ZERO NEUMANN OR DIRICHLET BOUNDARY CONDITIONS
C** CAN BE SOLVED BY SUITABLY MODIFYING THE BOUNDARY SOURCE TERMS.
C** (I.E. MODIFY S(I,J) WHERE I=1, OR I=NX, OR J=1, OR J=NY)
C**

```

```

C**  MUST CALL CMNUUH FIRST TO SET UP CONSTANTS REQUIRED BY SMNUUH.  **
C**  IF NONE OF (NX,NY,DAX,DAY,DAC) CHANGE, THEN SMNUUH MAY BE    **
C**  CALLED REPEATEDLY.  IF ANY OF THESE CHANGE, CMNUUH MUST BE   **
C**  CALLED TO RECALCULATE THE REQUIRED CONSTANTS.  IF TWO OR MORE  **
C**  EQUATIONS ARE SOLVED REPEATEDLY, ONE AFTER THE OTHER, THEN ONLY **
C**  ONE CALL TO CMNUUH FOR EACH IS REQUIRED IF THEY ARE ALLOCATED **
C**  DISTINCT CONSTANT ARRAYS, Q.                                  **
C**                                                                **
C*****
C*****
C
C   WORK IS DONE BY SUBROUTINE SMNUU1.
C
C   MX=(NX+1)/2
C   IW=MX*NY+1
C   CALL SMNUU1(S,H,NDX,NX,NY, Q,LQ, W1,W1(IW),W2,MX,W2(IW),MX-1)
C   RETURN
C   END OF SMNUUH.
C   END

```

```

SUBROUTINE CMNSUH(DAX, DAY, DAC, NDX, NX, NY, Q, LQ, W)
INTEGER NDX, NX, NY, LQ
DOUBLE PRECISION DAX(3, NX), DAY(3, 1), DAC
DOUBLE PRECISION W
DIMENSION Q(LQ), W(NY, 5)

C
C*****
C*****
C**
C**          INTRODUCTION:
C**
C**          COMPUTES THE CONSTANTS REQUIRED BY SMNSUH TO SOLVE THE
C**          FOLLOWING ( MIXED-NEUMANN ) HELMHOLTZ'S EQUATION ON
C**          AN NX*NY MESH.
C**
C**          FOR I = 1, ..., NX AND J = 1, ..., NY
C**
C**              AN( J ) * H( I, J+1 ) +
C**              AS( J ) * H( I, J-1 ) +
C**              AE( I ) * H( I+1, J ) +
C**              AW( I ) * H( I-1, J ) +
C**              AP( I, J ) * H( I, J ) = S( I, J )
C**
C**          WHERE
C**              AN( J ) = DAY( 3, 1 ) J=1...NY-1,
C**                      0 J= NY.
C**              AS( J ) = 0 J= 1.
C**                      DAY( 3, 1 ) J=2...NY,
C**              AE( I ) = DAX( 3, I ) I=1...NX-1,
C**                      0 I= NX.
C**              AW( I ) = 0 I= 1.
C**                      DAX( 1, I ) I=2...NX,
C**              AP( I, J ) = DAC+DAX( 2, 1 )-(AN( J )+AS( J )+BE( I )+BW( I ))
C**                      I=1...NX, J=1...NY.
C**              BE( I ) = DAX( 3, I ) I=1...NX.
C**              BW( I ) = DAX( 1, I ) I=1...NX.
C**
C**          USUALLY DAX( 2, I )=0.00 .
C**
C**          FOR DIRICHLET-NEUMANN PROBLEMS DAX( 1, 1 ), DAX( 3, NX ) ARE
C**                      SIMILAR TO OTHER DAX TERMS.
C**          FOR NEUMANN-NEUMANN PROBLEMS DAX( 1, 1 )=DAX( 3, NX )=0.00 .
C**
C**          FOR COMPATABILITY WITH OTHER SOLVERS SET DAY( 1, 1 )=0.00
C**                      AND DAY( 2, 1 )=0.00 .
C**
C*****
C**          ARGUMENT LIST: SEE OTHER COMMENTS FOR DETAILS
C**
C**          DAX, DAY, DAC - HELMHOLTZ COEFFICIENTS
C**
C**          NDX - 1ST ARRAY DIMENSION OF S AND H IN THE SOLVER
C**
C**          NX, NY - DIMENSIONS OF MESH (NY=P*2**L - SEE SMNSUH)
C**
C**          Q( LQ ) - ARRAY FOR REQUIRED CONSTANTS
C**
C**          LQ - SIZE OF Q, MUST BE AT LEAST LQ1+LQ2, WHERE:
C**                  LQ1 = NY + 10*LQ3**2 + 1
C**                  LQ2 = NX*(NY+3)
C**                  LQ3 = 0 IF P = 4, 5, OR 6;
C**                      (P-1)/2 IF P = 7 OR GREATER
C**

```

```

C**                                                     **
C**  W(NY,5)      -  DOUBLE PRECISION WORKSPACE ARRAY  **
C**                                                     **
C*****
C**                                                     **
C**          SPECIAL FEATURES OR NON-ANSI USAGE          **
C**                                                     **
C**  AN ERROR MESSAGE WILL BE OUTPUT TO CHANNEL 6, AND THE PROGRAM **
C**  TERMINATED, IF ANY OF THE FOLLOWING CONDITIONS ARE DETECTED: **
C**      NX              LESS THAN 3                      **
C**      NY              INCOMPATIBLE WITH THE FOURIER PACKAGE **
C**      LQ              TOO SMALL                        **
C**      DAX, DAY, DAC   NOT CONSISTANT WITH HELMHOLTZ'S EQUATION **
C**                                                     **
C**  THE WORKSPACE ARRAY, W, NEED NOT BE DISTINCT FROM THE **
C**  WORKSPACE SUPPLIED TO THE SOLVER.  HOWEVER THE LATTER MAY **
C**  CONSIST OF SINGLE PRECISION ARRAYS.                  **
C**                                                     **
C**  FOR MORE DETAILED DOCUMENTATION  -  SEE SMNSUH.      **
C**                                                     **
C*****
C*****
C

```

.  
 .  
 .  
 .

```

C  END OF CMNSUH.
C  END

```

```

SUBROUTINE SMNSUH(S,H,NDX,NX,NY, Q,LQ, W1,W2)
INTEGER NDX,NX,NY, LQ
DIMENSION S(NDX,NY),H(NDX,NY)
DIMENSION Q(LQ)
DIMENSION W1(NDX,NY),W2(NDX,NY)

```

```

C
C*****
C*****
C**
C**          DATE:          SEPTEMBER 1978
C**
C**          AUTHOR:        D.R. MOORE,  A.J. WALLCRAFT
C**
C**          ARCHIVIST:     A.J. WALLCRAFT
C**
C**          ADDRESS:       NORDA
C**                        CODE 322
C**                        NSTL STATION, MS 39529
C**
C**          PHONE:        601-688-4835  (COMMERCIAL)
C**                        494-4835      (ETS)
C**                        485-4835      (AUTOVON)
C**
C**          DATE OF CURRENT SEQUENCE:    DECEMBER 1982
C**
C*****
C**
C**          INTRODUCTION:
C**
C**          SOLVES FINITE DIFFERENCE HELMHOLTZ'S EQUATIONS:
C**          OVER A RECTANGLE IN CARTESIAN COORDINATES,
C**          ON A STRETCHED-UNIFORM MESH,
C**          WITH (ZERO) NEUMANN BOUNDARY CONDITIONS 1/2 GRID DISTANCE
C**          OUTSIDE THE MESH AT J=1 AND J=NY,
C**          AND EITHER (ZERO) NEUMANN BOUNDARY CONDITIONS 1/2 GRID
C**          DISTANCE OUTSIDE THE MESH OR (ZERO) DIRICHLET BOUNDARY
C**          CONDITIONS 1 GRID DISTANCE OUTSIDE THE MESH AT I=1 AND I=NX.
C**
C**          OR ANY EQUIVALENT SET OF EQUATIONS; FOR EXAMPLE:
C**          HELMHOLTZ'S EQUATION IN (SOME) NON-CARTESIAN COORDINATE
C**          SYSTEMS, OR MORE GENERAL (SEPARABLE) ELLIPTIC EQUATIONS.
C**
C**          USES THE FACR(0) METHOD.
C**
C**          BOTH THE INPUT ARRAY S(NDX,NY) AND THE OUTPUT ARRAY H(NDX,NY)
C**          ARE ASSUMED TO CONTAIN ACTIVE SUBARRAYS FROM 1 TO NX AND
C**          1 TO NY.  THE NON-ACTIVE SUBARRAY, NX+1 TO NDX AND 1 TO NY,
C**          OF S CAN BE ARBITRARY ON INPUT AND THAT OF H IS      RETURNED
C**          INTACT.
C**
C**          IN THE CALLING SEQUENCE THE INPUT AND OUTPUT ARRAYS MAY BE THE
C**          SAME ARRAY.  IF THEY ARE NOT THE SAME, THE INPUT ARRAY IS
C**          RETURNED INTACT BY THE SOLVER.
C**
C**          IT IS ASSUMED THAT NECESSARY CONSTANTS HAVE BEEN PRECOMPUTED
C**          EXTERNALLY AND ARE STORED IN THE ARRAY Q(LQ), WHICH
C**          IMPLICITLY DEFINES THE HELMHOLTZ COEFFICIENTS (DAX, DAY, DAC)
C**          TO THE SOLVER.
C**
C**          FOR I = 1, ..., NX  AND  J = 1, ..., NY  THE SOLUTION WILL
C**          SATISFY:
C**
C**

```

```

C**          AN( J) * H(I, J+1) +          **
C**          AS( J) * H(I, J-1) +          **
C**          AE(I ) * H(I+1,J ) +          **
C**          AW(I ) * H(I-1,J ) +          **
C**          AP(I,J) * H(I, J ) = S(I,J)    **
C**
C** WHERE
C**      AN(J)  = DAY(3,1)  J=1...NY-1,      **
C**              0          J= NY.           **
C**      AS(J)  = 0          J= 1.           **
C**              DAY(3,1)  J=2...NY,         **
C**      AE(I)  = DAX(3,I)  I=1...NX-1,      **
C**              0          I= NX.           **
C**      AW(I)  = 0          I= 1.           **
C**              DAX(1,I)  I=2...NX,         **
C**      AP(I,J) = DAC+DAX(2,1)-(AN(J)+AS(J)+BE(I)+BW(I))
C**                                     I=1...NX, J=1...NY. **
C**      BE(I)  = DAX(3,I)  I=1...NX.       **
C**      BW(I)  = DAX(1,I)  I=1...NX.       **
C**
C*****
C**
C**      LIBRARIES REQUIRED:
C**
C**      FAST FOURIER TRANSFORM LIBRARY      BY D.R. MOORE
C**      TRIDIAGONAL LINEAR EQUATION SOLVER LIBRARY BY A.J. WALLCRAFT
C**
C*****
C**
C**      ARGUMENT LIST:  SEE OTHER COMMENTS FOR DETAILS
C**
C**      S(NDX,NY)  -  INPUT ARRAY
C**
C**      H(NDX,NY)  -  OUTPUT ARRAY
C**
C**      NDX,NY     -  SIZES OF S AND H
C**
C**      NX         -  ACTIVE SUBDIMENSION OF S AND H
C**
C**      Q(LQ)      -  CONSTANT ARRAY, PRECOMPUTED EXTERNALLY
C**
C**      LQ         -  DIMENSION OF Q
C**
C**      W1(NDX,NY) -  WORKSPACE ARRAYS,
C**      W2(NDX,NY)   RETURNED AND AVAILABLE AFTER EACH CALL.
C**
C*****
C**
C**      SPECIAL FEATURES OR NON-ANSI USAGE:
C**
C**      DOUBLE PRECISION LIBRARIES USE THE IMPLICIT STATEMENT
C**
C**      TI-ASC ONLY:  USE W-OPTION WHEN COMPILING FOURIER PACKAGES
C**
C**      NX IS ARBITRARY (.GE.3).
C**      NY MUST BE OF THE FORM P*2**L,    WITH L = 1, ..., 7.
C**      USUALLY P=4, 5, OR 6 -
C**      HIGHER VALUES OF P ARE POSSIBLE, BUT ARE LESS EFFICIENT.
C**      LQ MUST BE AT LEAST LQ1+LQ2 (SEE CMNSUH).
C**
C**      PROBLEMS WITH NON-ZERO NEUMANN OR DIRICHLET BOUNDARY CONDITIONS
C**      CAN BE SOLVED BY SUITABLY MODIFYING THE BOUNDARY SOURCE TERMS.
C**      (I.E. MODIFY S(I,J) WHERE I=1, OR I=NX, OR J=1, OR J=NY)

```

```

C**                                **
C** MUST CALL CMNSUH FIRST TO SET UP CONSTANTS REQUIRED BY SMNSUH. **
C** IF NONE OF (NX,NY,DAX,DAY,DAC) CHANGE, THEN SMNSUH MAY BE **
C** CALLED REPEATEDLY. IF ANY OF THESE CHANGE, CMNSUH MUST BE **
C** CALLED TO RECALCULATE THE REQUIRED CONSTANTS. IF TWO OR MORE **
C** EQUATIONS ARE SOLVED REPEATEDLY, ONE AFTER THE OTHER. THEN ONLY **
C** ONE CALL TO CMNSUH FOR EACH IS REQUIRED IF THEY ARE ALLOCATED **
C** DISTINCT CONSTANT ARRAYS, Q. **
C**                                **
C*****
C*****
C
C FORWARD FFT.
C
C CALL COSODD(S,W1, W2,Q,NDX,NX,NY)
C
C SOLVE RESULTING TRIDIAGONAL SYSTEMS.
C
C   IQT=INT(Q(1))+1
C   IQQ=IQT + 3*NX
C   CALL TRANSP(W1, NDX,NX,NY, W2,NY)
C   CALL TGAUSS(W2,W2,NY, NY,NX, Q(IQT),Q(IQQ))
C   CALL TRANSP(W2, NY, NY,NX, W1,NDX)
C
C REVERSE FFT.
C
C DO 11 I=1,NX
C   W1(I,1)=0.5*W1(I,1)
11 CONTINUE
C   CALL COSODI(W1,H, W2,Q,NDX,NX,NY)
C   RETURN
C END OF SMNSUH.
END

```



```

SUBROUTINE CMSSH(DAX, DAY, DAC, NDX, NX, NY, QX, LOX, QY, LOY, W)
INTEGER NDX, NX, NY, LOX, LOY
DOUBLE PRECISION DAX(3, NX), DAY(3, NY), DAC
DOUBLE PRECISION W
DIMENSION QX(NX, NX, 2), QY(LOY), W(NX, 1)
DIMENSION W(NX, NY+5)
C
C
C*****
C*****
C**
C**      INTRODUCTION:
C**
C**      COMPUTES THE CONSTANTS REQUIRED BY SMSSH TO SOLVE THE
C**      FOLLOWING ( MIXED-MIXED ) HELMHOLTZ'S EQUATION ON
C**      AN NX*NY MESH.
C**
C**      FOR I = 1, ..., NX AND J = 1, ..., NY
C**
C**          AN( J ) * H( I, J+1 ) +
C**          AS( J ) * H( I, J-1 ) +
C**          AE( I ) * H( I+1, J ) +
C**          AW( I ) * H( I-1, J ) +
C**          AP( I, J ) * H( I, J ) = S( I, J )
C**
C**      WHERE
C**          AN( J ) = DAY( 3, J ) J=1...NY-1,
C**                   0 J= NY.
C**          AS( J ) = 0 J= 1.
C**                   DAY( 1, J ) J=2...NY,
C**          AE( I ) = DAX( 3, I ) I=1...NX-1,
C**                   0 I= NX.
C**          AW( I ) = 0 I= 1.
C**                   DAX( 1, I ) I=2...NX,
C**          AP( I, J ) = DAC+DAX( 2, 1 )-(BN( J )+BS( J )+BE( I )+BW( I ))
C**                                     I=1...NX, J=1...NY.
C**          BN( J ) = DAY( 3, J ) J=1...NY.
C**          BS( J ) = DAY( 1, J ) J=1...NY.
C**          BE( I ) = DAX( 3, I ) I=1...NX.
C**          BW( I ) = DAX( 1, I ) I=1...NX.
C**
C**      USUALLY DAX( 2, 1 )=0.DO AND DAY( 2, J )=0.DO.
C**
C**      FOR DIRICHLET-MIXED PROBLEMS DAX( 1, 1 ), DAX( 3, NX ) ARE
C**                                SIMILAR TO OTHER DAX TERMS.
C**      FOR NEUMANN-MIXED PROBLEMS DAX( 1, 1 )=DAX( 3, NX )=0.DO .
C**
C**      FOR MIXED-DIRICHLET PROBLEMS DAY( 1, 1 ), DAY( 3, NY ) ARE
C**                                SIMILAR TO OTHER DAY TERMS.
C**      FOR MIXED-NEUMANN PROBLEMS DAY( 1, 1 )=DAY( 3, NY )=0.DO .
C**
C**
C*****
C**
C**      ARGUMENT LIST: SEE OTHER COMMENTS FOR DETAILS
C**
C**      DAX, DAY, DAC - HELMHOLTZ COEFFICIENTS
C**
C**      NDX - 1ST ARRAY DIMENSION OF S AND H IN THE SOLVER
C**
C**      NX, NY - DIMENSIONS OF MESH
C**
C**      QX( NX, NX, 2 ) - ARRAY FOR REQUIRED CONSTANTS (SEE 'USAGE' BELOW)
C**
C**

```

```

C** QY(LQY)      -  ARRAY FOR REQUIRED CONSTANTS                **
C**                                                     **
C** LQX          -  SIZE OF QX, MUST BE AT LEAST  2*NX*NX      **
C**                                                     **
C** LQY          -  SIZE OF QY, MUST BE AT LEAST  NY*(NX+3)    **
C**                                                     **
C** W(NX,NY+5)   -  DOUBLE PRECISION WORKSPACE ARRAY          **
C**                                                     **
C*****
C**
C**          SPECIAL FEATURES OR NON-ANSI USAGE                **
C**                                                     **
C** QX IS USED AS (DOUBLE PRECISION) WORKSPACE IN 'ESYSON'. IT IS **
C** THEREFORE ADVISABLE TO EQUIVALENCE QX(1,1,1) TO A          **
C** DOUBLE PRECISION VARIABLE.                                  **
C**                                                     **
C** AN ERROR MESSAGE WILL BE OUTPUT TO CHANNEL 6, AND THE PROGRAM **
C** TERMINATED, IF ANY OF THE FOLLOWING CONDITIONS ARE DETECTED: **
C**      NX              LESS THAN 3                          **
C**      NY              LESS THAN 4                          **
C**      NY              LESS THAN NX                          **
C**      LQX,LQY         TOO SMALL                             **
C**      DAX,DAY,DAC     NOT CONSISTANT WITH HELMHOLTZ'S EQUATION **
C**                                                     **
C** THE WORKSPACE ARRAY, W, NEED NOT BE DISTINCT FROM THE      **
C** WORKSPACE SUPPLIED TO THE SOLVER. HOWEVER THE LATTER MAY   **
C** CONSIST OF SINGLE PRECISION ARRAYS.                         **
C**                                                     **
C** FOR MORE DETAILED DOCUMENTATION -  SEE SMMSSH.             **
C**                                                     **
C*****
C*****
C
      .
      .
      .
      .
C
C      END OF CMMSSH.
C      END

```

```

SUBROUTINE SHMSSH(S,H,NDX,NX,NY, QX,LQX,QY,LQY, W1)
INTEGER NDX,NX,NY, LQX,LQY
DIMENSION S(NDX,NY),H(NDX,NY)
DIMENSION QX(NX,NX,2),QY(LQY)
DIMENSION W1(NDX,NY)

```

```

C
C*****
C*****
C**
C**          DATE:          DECEMBER 1980
C**
C**          AUTHOR:        A.J. WALLCRAFT
C**
C**          ARCHIVIST:     A.J. WALLCRAFT
C**
C**          ADDRESS:       NORDA
C**                        CODE 322
C**                        NSTL STATION, MS 39529
C**
C**          PHONE:         601-688-4835 (COMMERCIAL)
C**                        494-4835 (FTS)
C**                        485-4835 (AUTOVON)
C**
C**          DATE OF CURRENT SEQUENCE:  DECEMBER 1982
C**
C*****
C**
C**          INTRODUCTION:
C**
C**          SOLVES FINITE DIFFERENCE HELMHOLTZ'S EQUATIONS:
C**          OVER A RECTANGLE,
C**          ON A STRETCHED-STRETCHED MESH,
C**          WITH EITHER (ZERO) NEUMANN BOUNDARY CONDITIONS 1/2 GRID
C**            DISTANCE OUTSIDE THE MESH OR (ZERO) DIRICHLET BOUNDARY
C**            CONDITIONS 1 GRID DISTANCE OUTSIDE THE MESH AT J=1 AND J=NY,
C**          AND EITHER (ZERO) NEUMANN BOUNDARY CONDITIONS 1/2 GRID
C**            DISTANCE OUTSIDE THE MESH OR (ZERO) DIRICHLET BOUNDARY
C**            CONDITIONS 1 GRID DISTANCE OUTSIDE THE MESH AT I=1 AND I=NX.
C**
C**          OR ANY EQUIVALENT SET OF EQUATIONS; FOR EXAMPLE:
C**            HELMHOLTZ'S EQUATION IN (SOME) NON-CARTESIAN COORDINATE
C**            SYSTEMS, OR MORE GENERAL (SEPARABLE) ELLIPTIC EQUATIONS.
C**
C**          USING THE MATRIX EIGENVALUE DECOMPOSITION METHOD.
C**
C**          BOTH THE INPUT ARRAY S(NDX,NY) AND THE OUTPUT ARRAY H(NDX,NY)
C**          ARE ASSUMED TO CONTAIN ACTIVE SUBARRAYS FROM 1 TO NX AND
C**          1 TO NY. THE NON-ACTIVE SUBARRAY, NX+1 TO NDX AND 1 TO NY,
C**          OF S CAN BE ARBITRARY ON INPUT AND THAT OF H IS RETURNED
C**          INTACT.
C**
C**          IN THE CALLING SEQUENCE THE INPUT AND OUTPUT ARRAYS MAY BE THE
C**          SAME ARRAY. IF THEY ARE NOT THE SAME, THE INPUT ARRAY IS
C**          RETURNED INTACT BY THE SOLVER.
C**
C**          IT IS ASSUMED THAT NECESSARY CONSTANTS HAVE BEEN PRECOMPUTED
C**          EXTERNALLY AND ARE STORED IN THE ARRAYS QX AND QY, WHICH
C**          IMPLICITLY DEFINES THE HELMHOLTZ COEFFICIENTS (DAX, DAY, DAC)
C**          TO THE SOLVER.
C**
C**
C**          FOR I = 1, ..., NX AND J = 1, ..., NY THE SOLUTION WILL
C**          SATISFY:

```

```

C**
C**          AN( J) * H(I, J+1) +
C**          AS( J) * H(I, J-1) +
C**          AE(I ) * H(I+1,J ) +
C**          AW(I ) * H(I-1,J ) +
C**          AP(I,J) * H(I, J ) = S(I,J)
C**
C** WHERE
C**      AN(J)  = DAY(3,J)  J=1...NY-1,
C**              0          J= NY.
C**      AS(J)  = 0          J= 1.
C**              DAY(1,J)  J=2...NY,
C**      AE(I)  = DAX(3,I)  I=1...NX-1,
C**              0          I= NX.
C**      AW(I)  = 0          I= 1.
C**              DAX(1,I)  I=2...NX,
C**      AP(I,J) = DAC+DAX(2,1)-(BN(J)+BS(J)+BE(I)+BW(I))
C**                                  I=1...NX, J=1...NY.
C**      BN(J)  = DAY(3,J)  J=1...NY
C**      BS(J)  = DAY(1,J)  J=1...NY
C**      BE(I)  = DAX(3,I)  I=1...NX.
C**      BW(I)  = DAX(1,I)  I=1...NX.
C**
C*****
C**
C**      LIBRARIES REQUIRED:
C**
C**      LINPACK BY
C**      TRIDIAGONAL LINEAR EQUATION SOLVER LIBRARY BY A.J. WALLCRAFT
C**      ROUTINE HQR2 IS FROM EISPACK
C**
C*****
C**
C**      ARGUMENT LIST:  SEE OTHER COMMENTS FOR DETAILS
C**
C**      S(NDX,NY)  -  INPUT ARRAY
C**
C**      H(NDX,NY)  -  OUTPUT ARRAY
C**
C**      NDX,NY     -  SIZES OF S AND H
C**
C**      NX         -  ACTIVE SUBDIMENSION OF S AND H
C**
C**      QX(NX,NX,2) -  CONSTANT ARRAY, PRECOMPUTED EXTERNALLY
C**
C**      QY(LQY)     -  CONSTANT ARRAY, PRECOMPUTED EXTERNALLY
C**
C**      LQX         -  DIMENSION OF QX
C**
C**      LQY         -  DIMENSION OF QY
C**
C**      W1(NDX,NY) -  WORKSPACE ARRAY,
C**                      RETURNED AND AVAILABLE AFTER EACH CALL.
C**
C*****
C**
C**      SPECIAL FEATURES OR NON-ANSI USAGE:
C**
C**      DOUBLE PRECISION LIBRARIES USE THE IMPLICIT STATEMENT
C**
C**      REPLACE MATRIX-MATRIX MULTIPLIES (BELOW) WITH FASTEST (MACHINE
C**      CODE) VERSION AVAILABLE FOR TARGET COMPUTER. THIS MAY INVOLVE
C**      USING THE TRANSPOSE OF QX - IF SO MODIFY CMMSSH APPROPRIATELY.

```

```

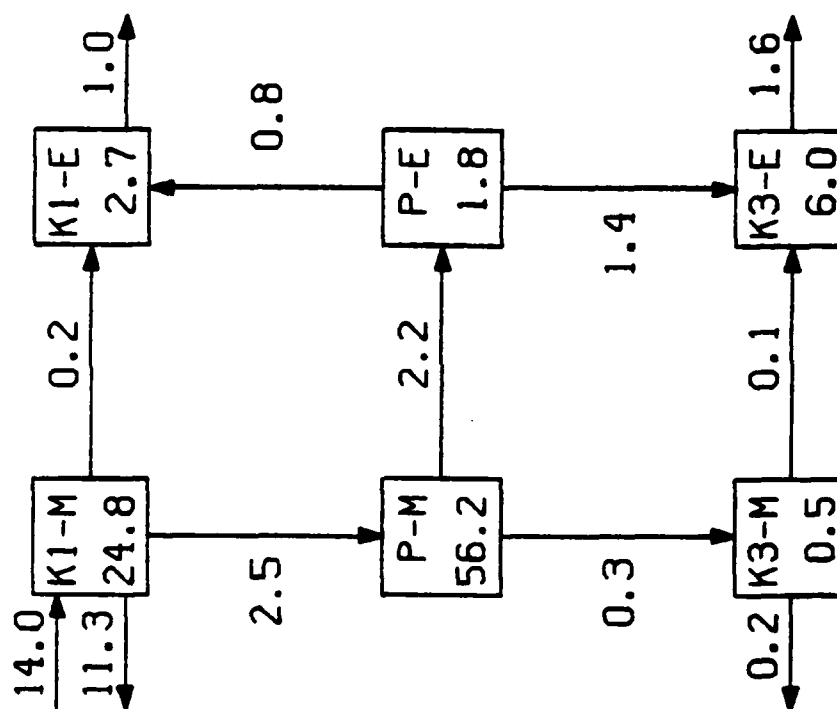
C**
C** NX IS ARBITRARY (.GE.3) BUT MUST NOT BE GREATER THEN NY. **
C** NY IS ARBITRARY (.GE.4). **
C** LQX MUST BE AT LEAST 2*NX*NX. **
C** LQY MUST BE AT LEAST NY*(NX+3). **
C**
C** PROBLEMS WITH NON-ZERO NEUMANN OR DIRICHLET BOUNDARY CONDITIONS **
C** CAN BE SOLVED BY SUITABLY MODIFYING THE BOUNDARY SOURCE TERMS. **
C** (I.E. MODIFY S(I,J) WHERE I=1, OR I=NX, OR J=1, OR J=NY) **
C**
C** MUST CALL CMMSSH FIRST TO SET UP CONSTANTS REQUIRED BY SMMSSH. **
C** IF NONE OF (NX,NY,DAX,DAY,DAC) CHANGE, THEN SMMSSH MAY BE **
C** CALLED REPEATEDLY. IF ANY OF THESE CHANGE, CMMSSH MUST BE **
C** CALLED TO RECALCULATE THE REQUIRED CONSTANTS. IF TWO OR MORE **
C** EQUATIONS ARE SOLVED REPEATEDLY, ONE AFTER THE OTHER, THEN ONLY **
C** ONE CALL TO CMMSSH FOR EACH IS REQUIRED IF THEY ARE ALLOCATED **
C** DISTINCT CONSTANT ARRAYS, QX,QY. IF TWO OR MORE EQUATIONS **
C** DIFFER ONLY IN (NY,DAY,AC), AND NOT IN (NX,AX). THEN THEY CAN **
C** SHARE A SINGLE QX ARRAY, ONLY DISTINCT QY ARRAYS ARE REQUIRED. **
C**
C*****
C*****
C
C FORWARD TRANSFORM (BY MATRIX-MATRIX MULTIPLY).
C
C DO 11 J=1,NY
C DO 11 I=1,NX
C W1(I,J)=0.0
C DO 11 K=1,NX
C W1(I,J)=W1(I,J) + QX(K,I,2)*S(K,J)
11 CONTINUE
C
C C SOLVE RESULTING TRIDIAGONAL SYSTEMS.
C
C IQY=3*NY+1
C CALL TGAUSS(W1,W1,NDX,NX,NY, QY,QY(IQY))
C
C C REVERSE TRANSFORM (BY MATRIX-MATRIX MULTIPLY).
C
C DO 21 J=1,NY
C DO 21 I=1,NX
C H(I,J)=0.0
C DO 21 K=1,NX
C H(I,J)=H(I,J) + QX(K,I,1)*W1(K,J)
21 CONTINUE
C RETURN
C END OF SMMSSH.
C END

```

## APPENDIX B

Example of Quasi-Geostrophic Regional Energetics Graphical Output

S. GYRE HOLLAND  
ENERGY BOX DIAGRAM FOR ( 2. 24 ) - ( 2. 24 )

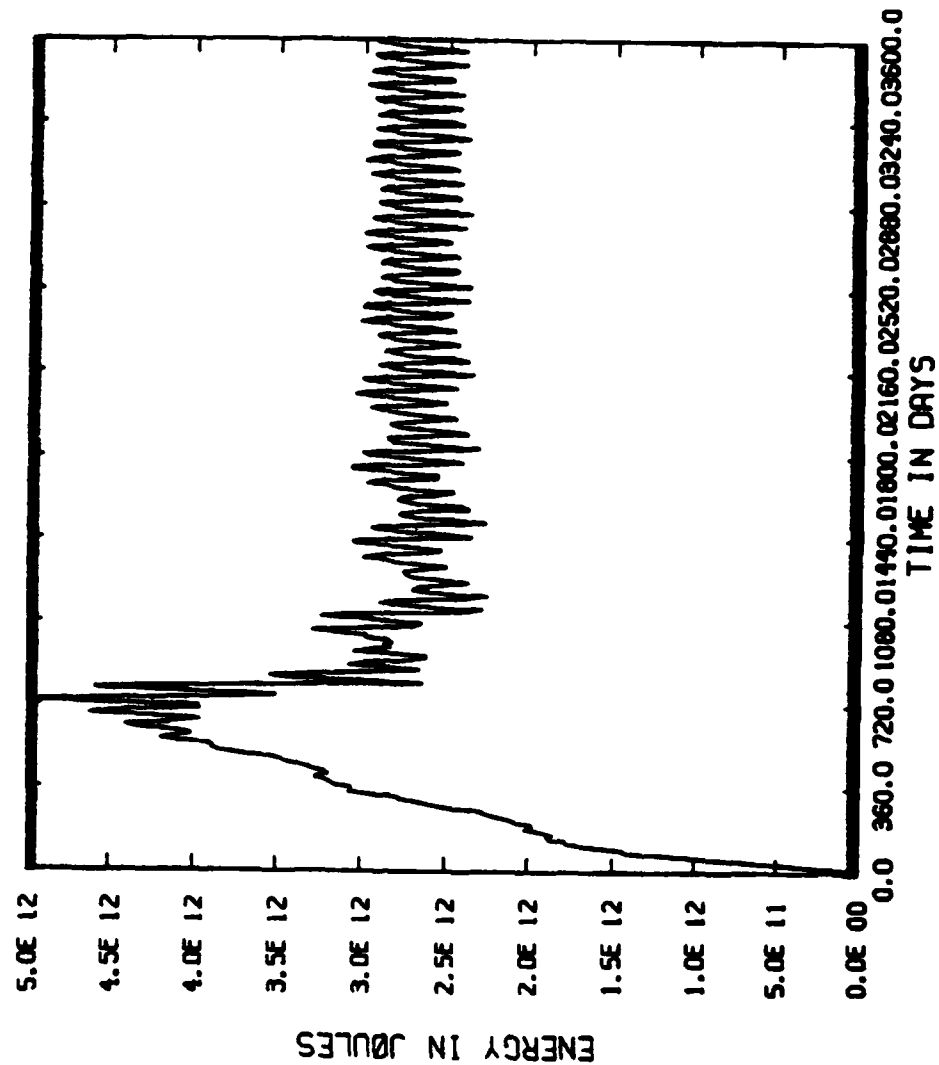


RESERVOIR UNITS - 1.0E 11 JOULES      TRANSFER UNITS - 1.0E 05 JOULES/SEC

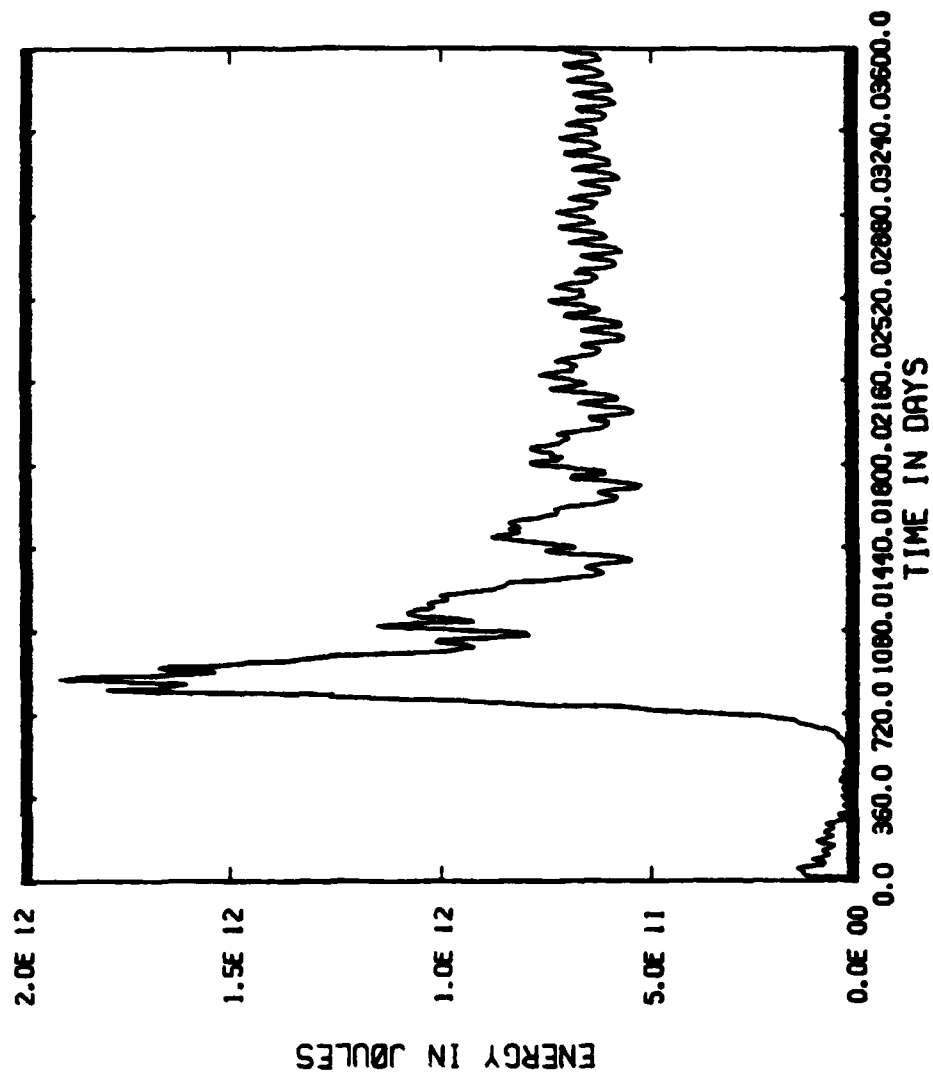




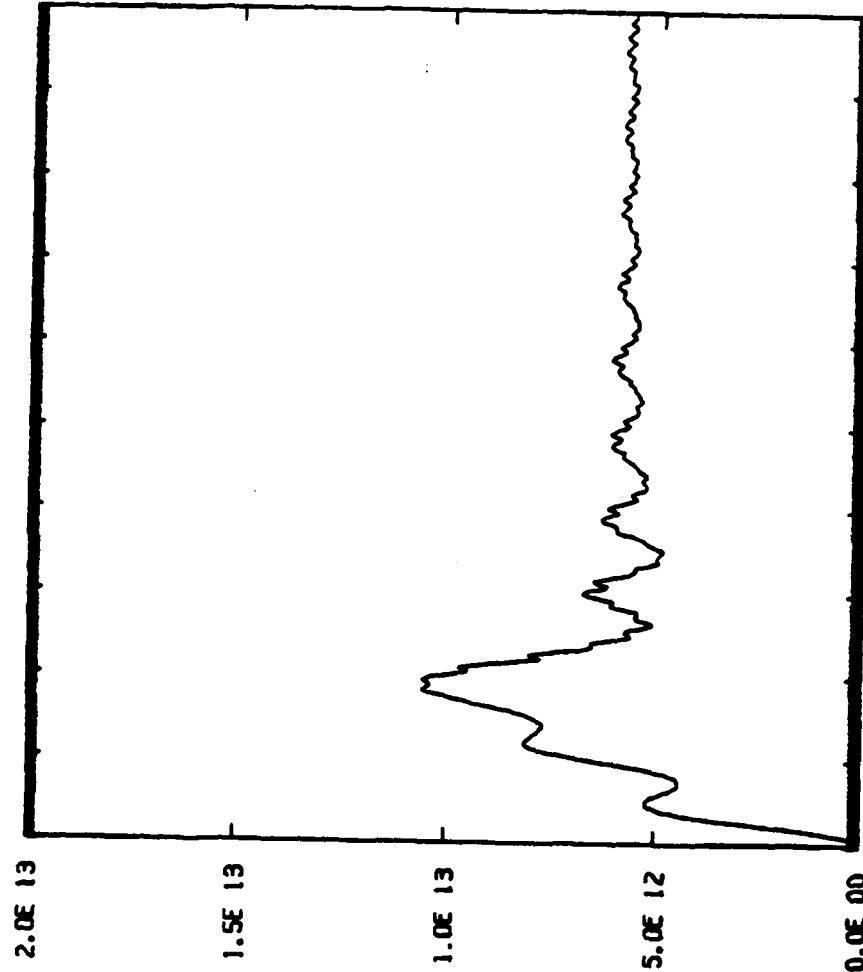
# KINETIC ENERGY (1) S. GYRE HØLLAND



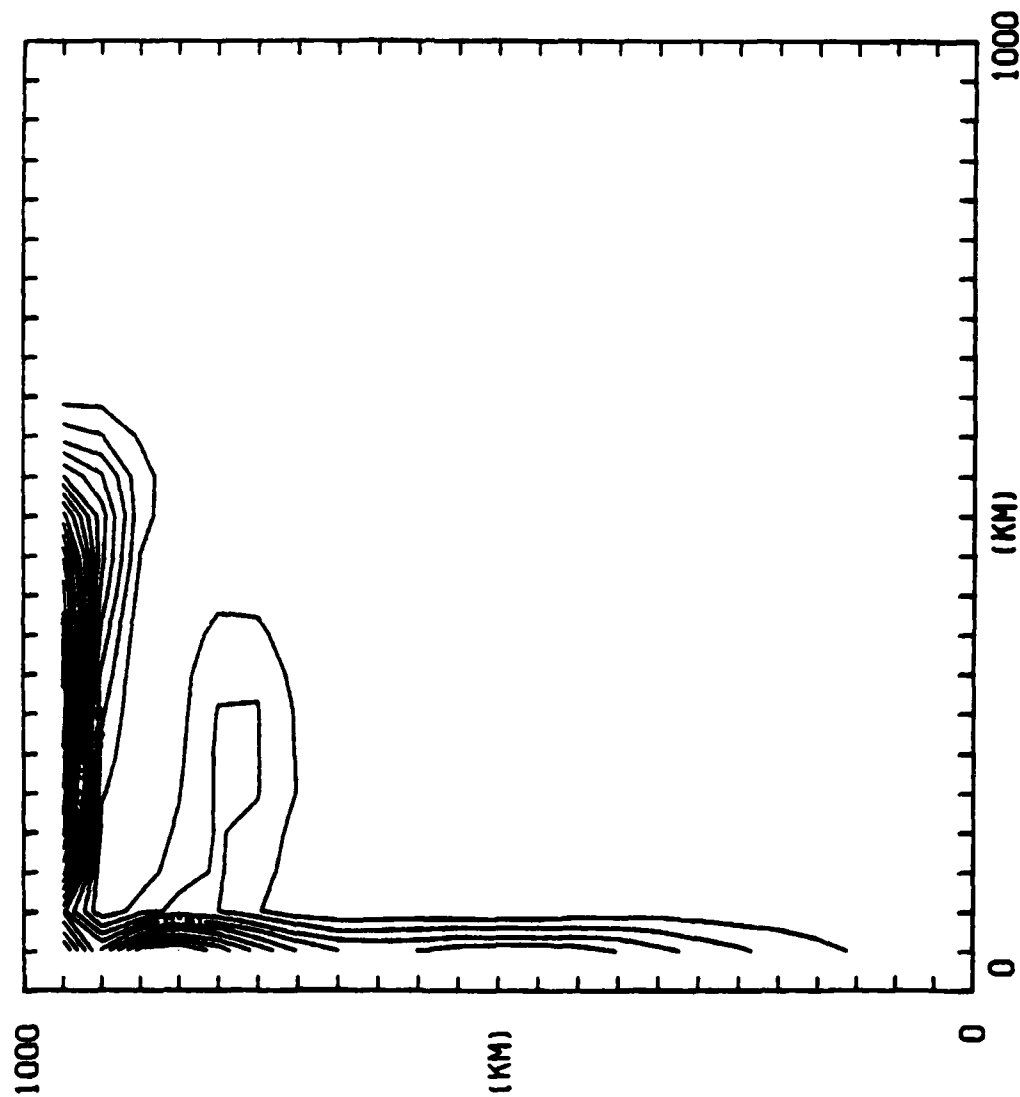
# KINETIC ENERGY (3) S. GYRE HOLLAND



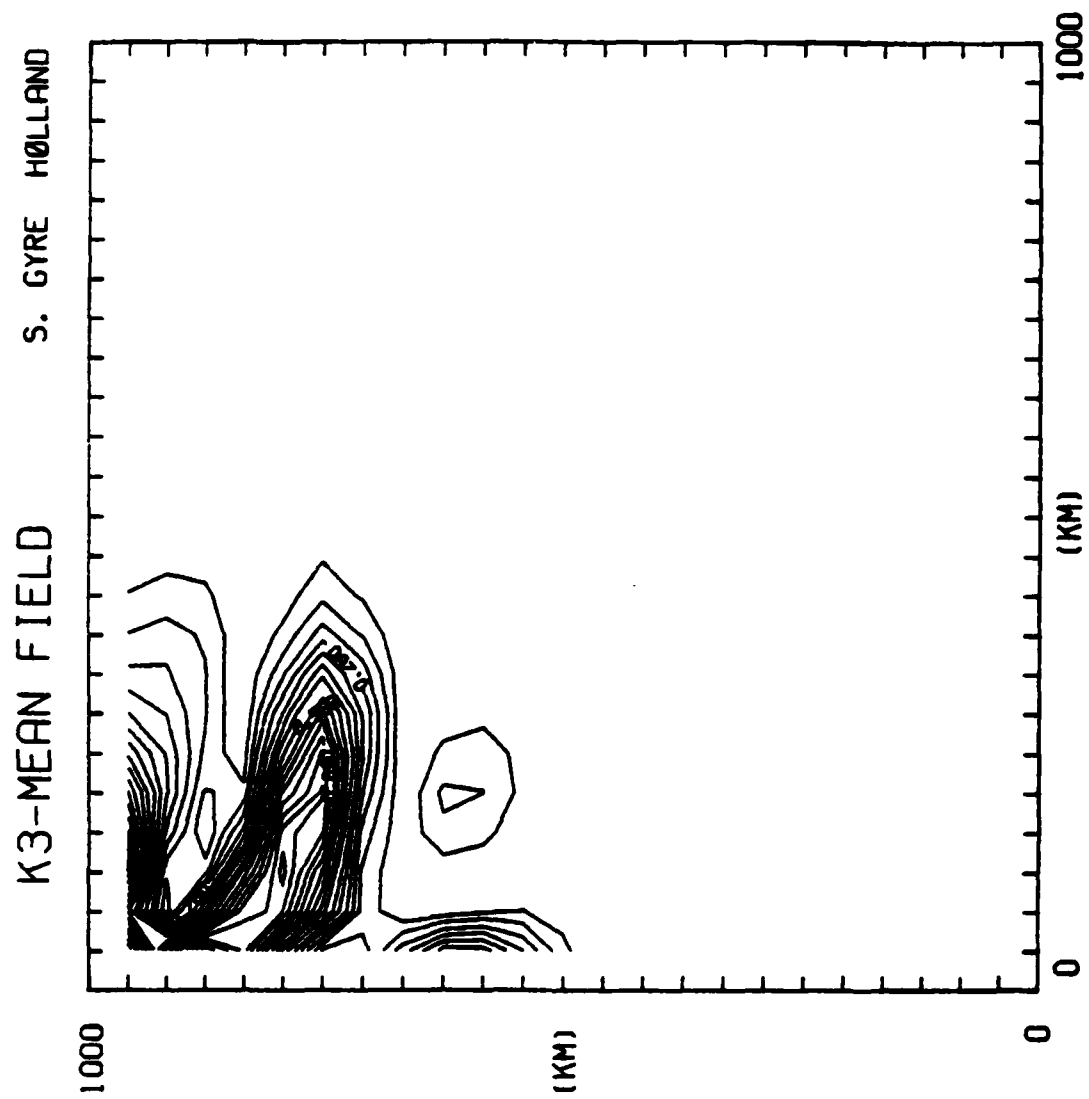
**S. GYRE HØLLAND**



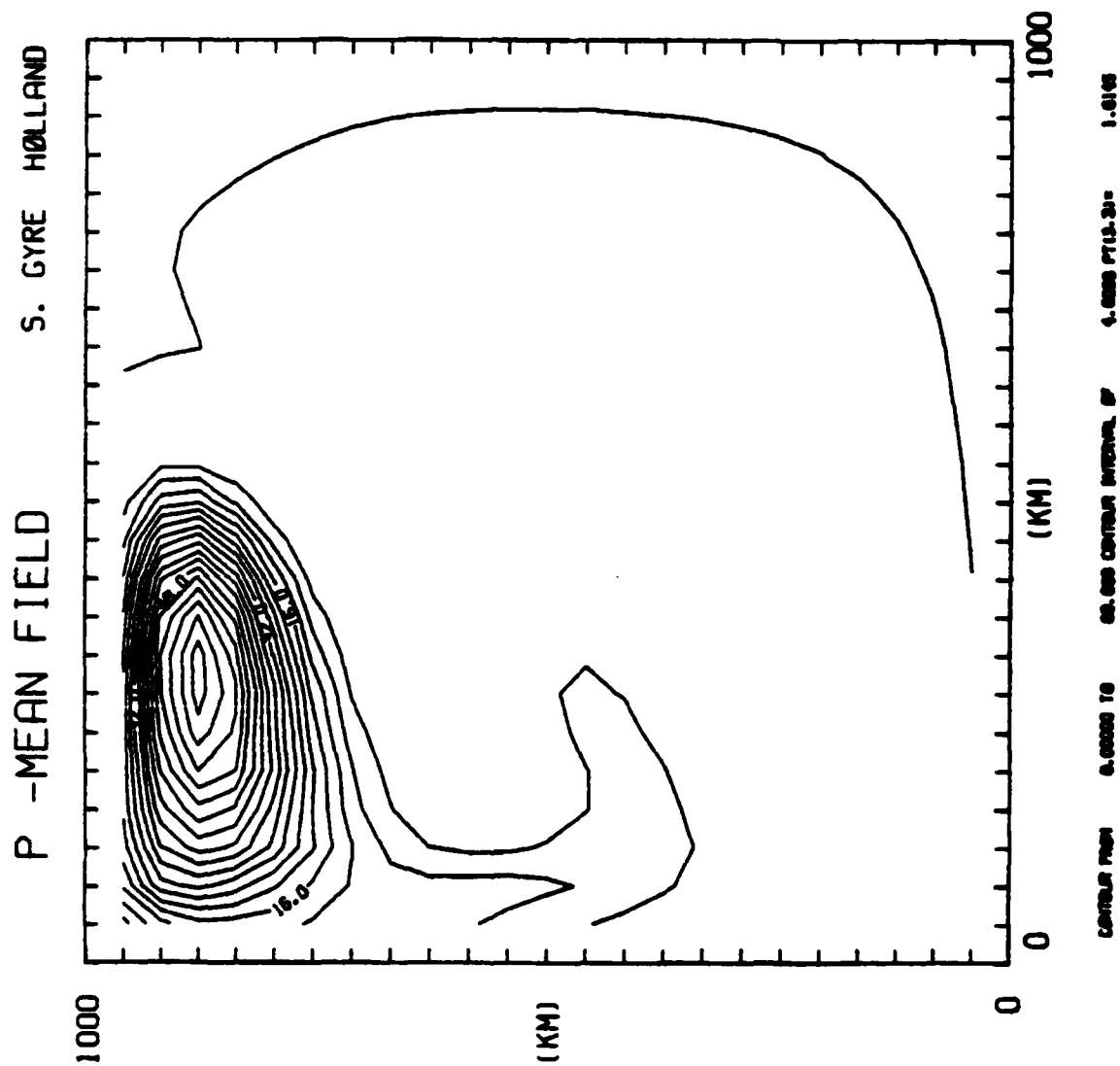
# K1-MEAN FIELD S. GYRE HOLLAND



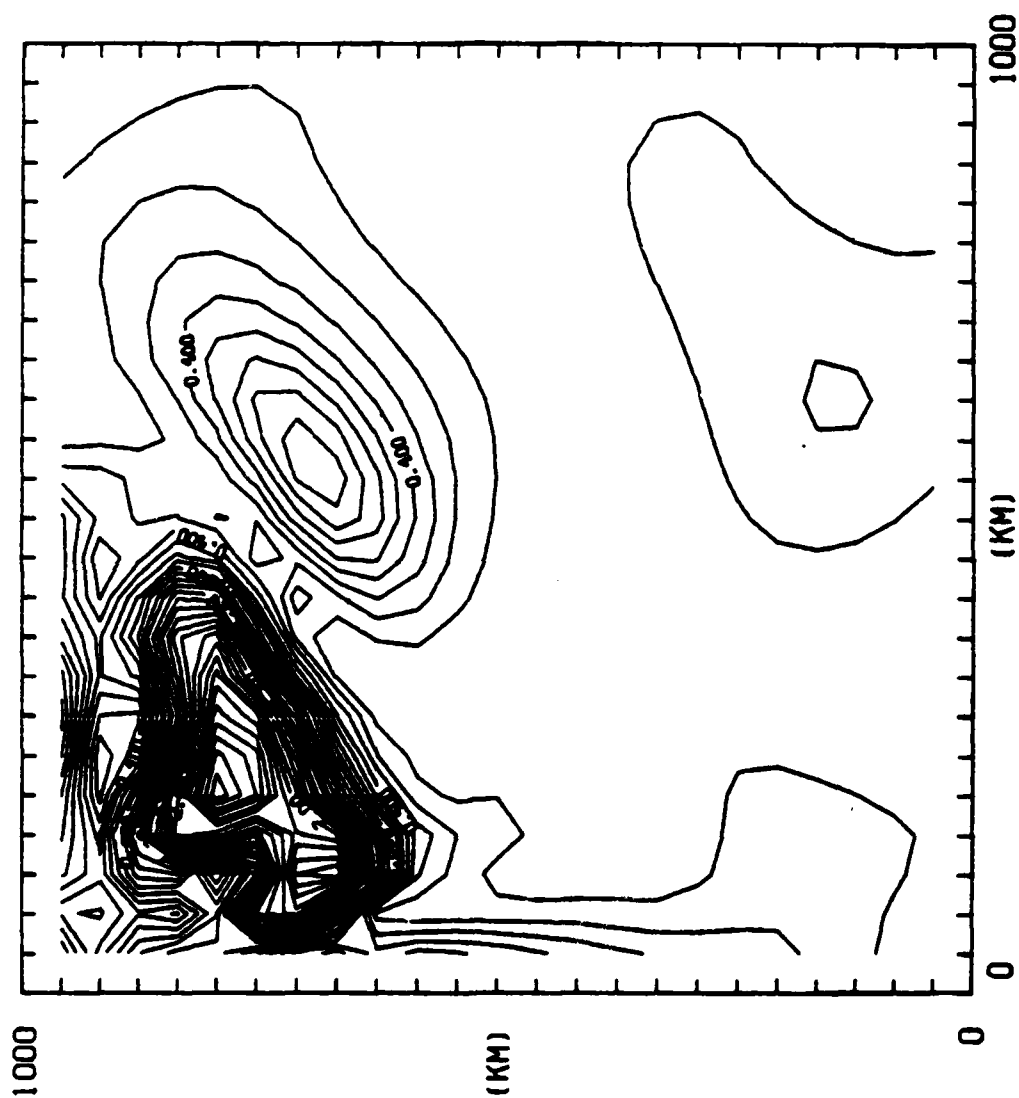
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CONTOUR FROM 0.00000 TO 1.1000 CONTOUR INTERVAL OF 0.10000E-01 PT10.30= 0.10001E-02

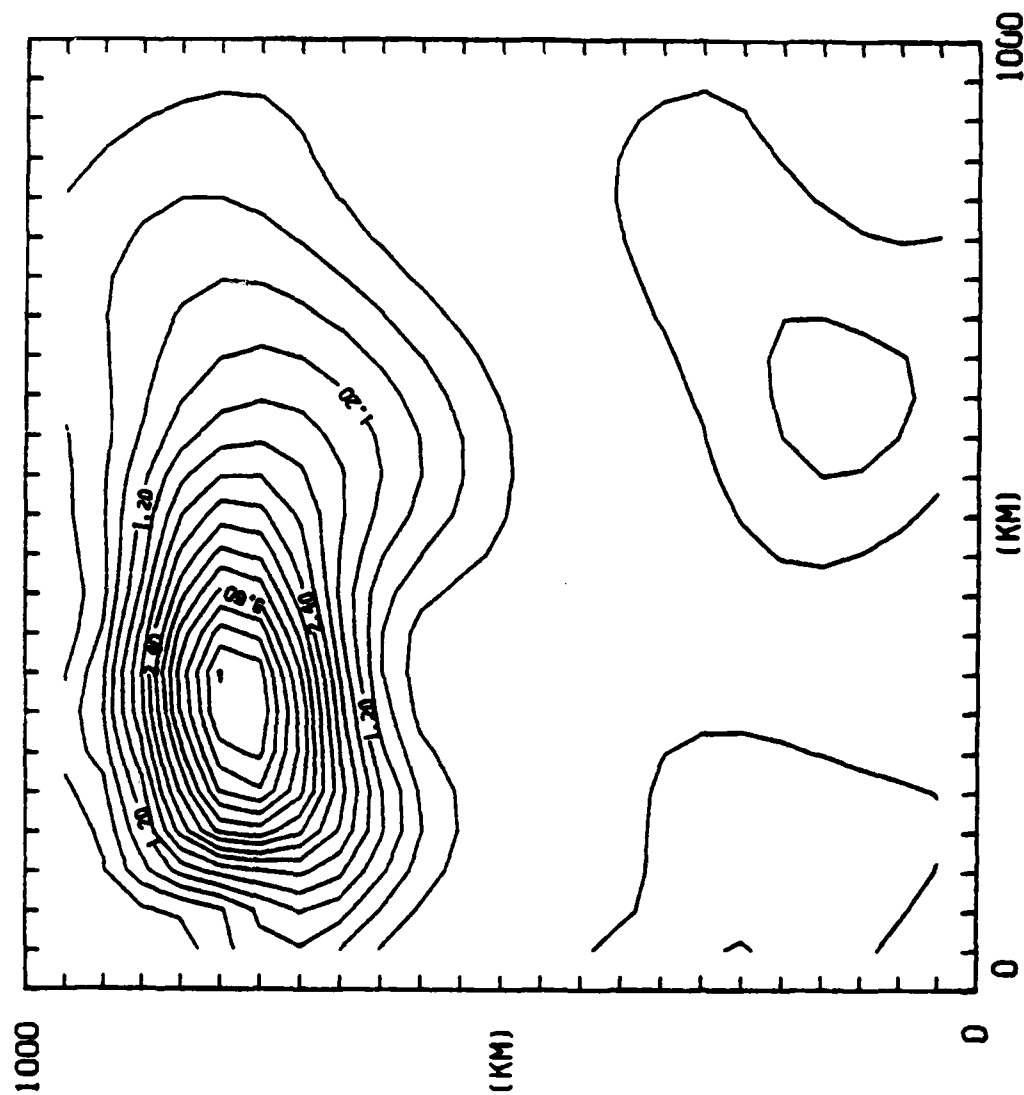


# K1-EDDY FIELD      S. GYRE HØLLAND



CONTOUR FROM 0.0000 TO 2.7000 CONTOUR INTERVAL OF 0.100000 00 P113.31+ 0.00007E-01

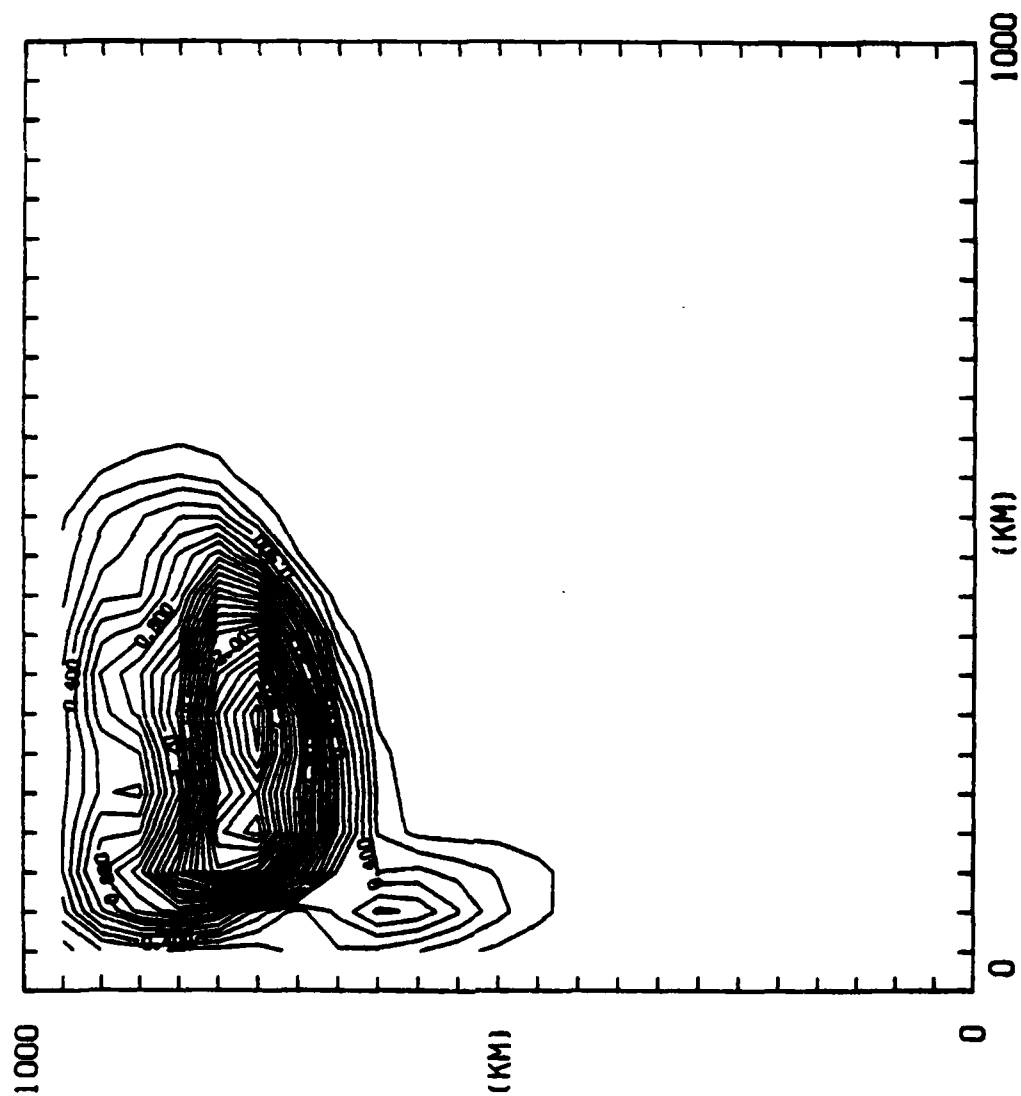
# K3-EDDY FIELD      S. GYRE HØLLAND



CONTOUR FROM 0.0000 TO 0.0000 CONTOUR INTERVAL OF 0.0000 P113.31- 0.30206

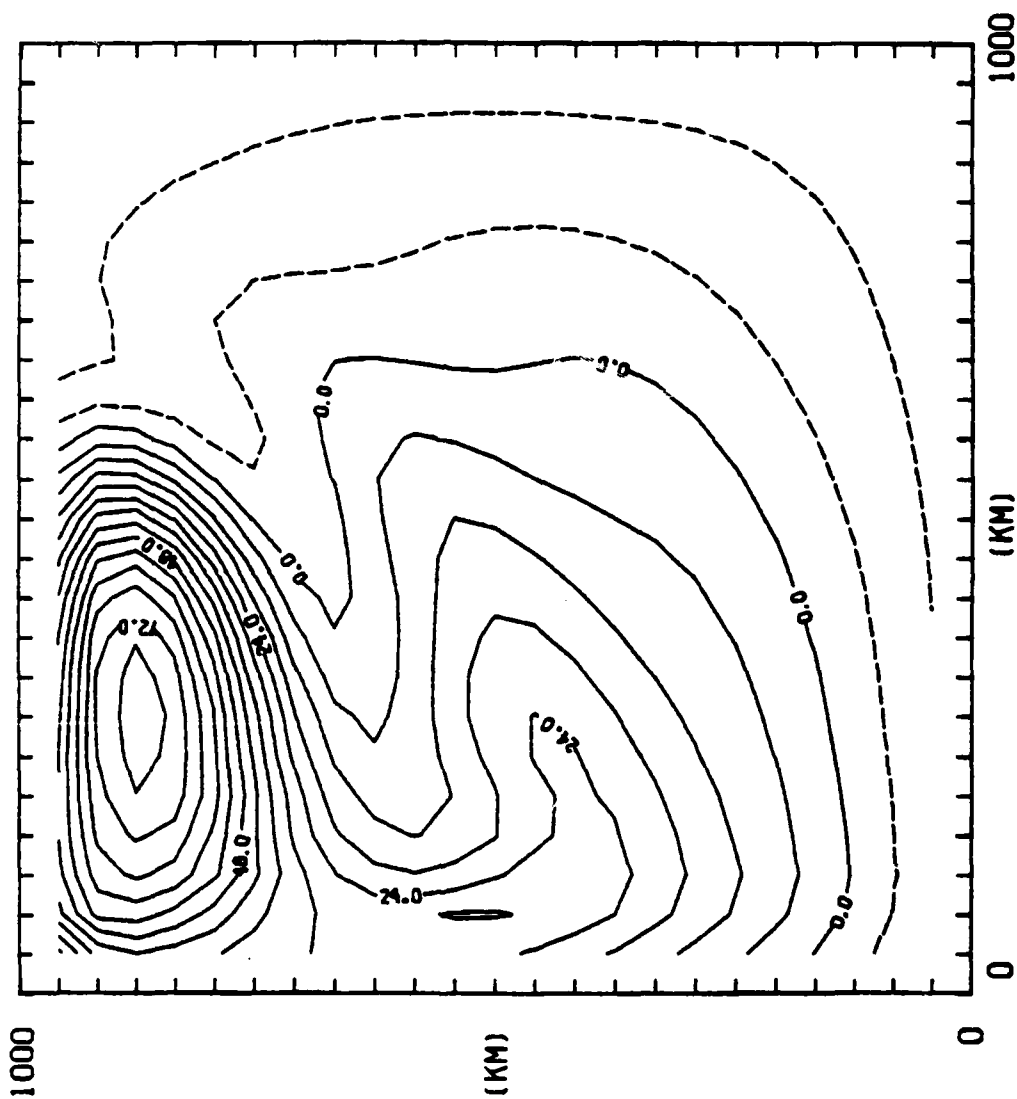


# P -EDDY FIELD      S. GYRE HØLLAND



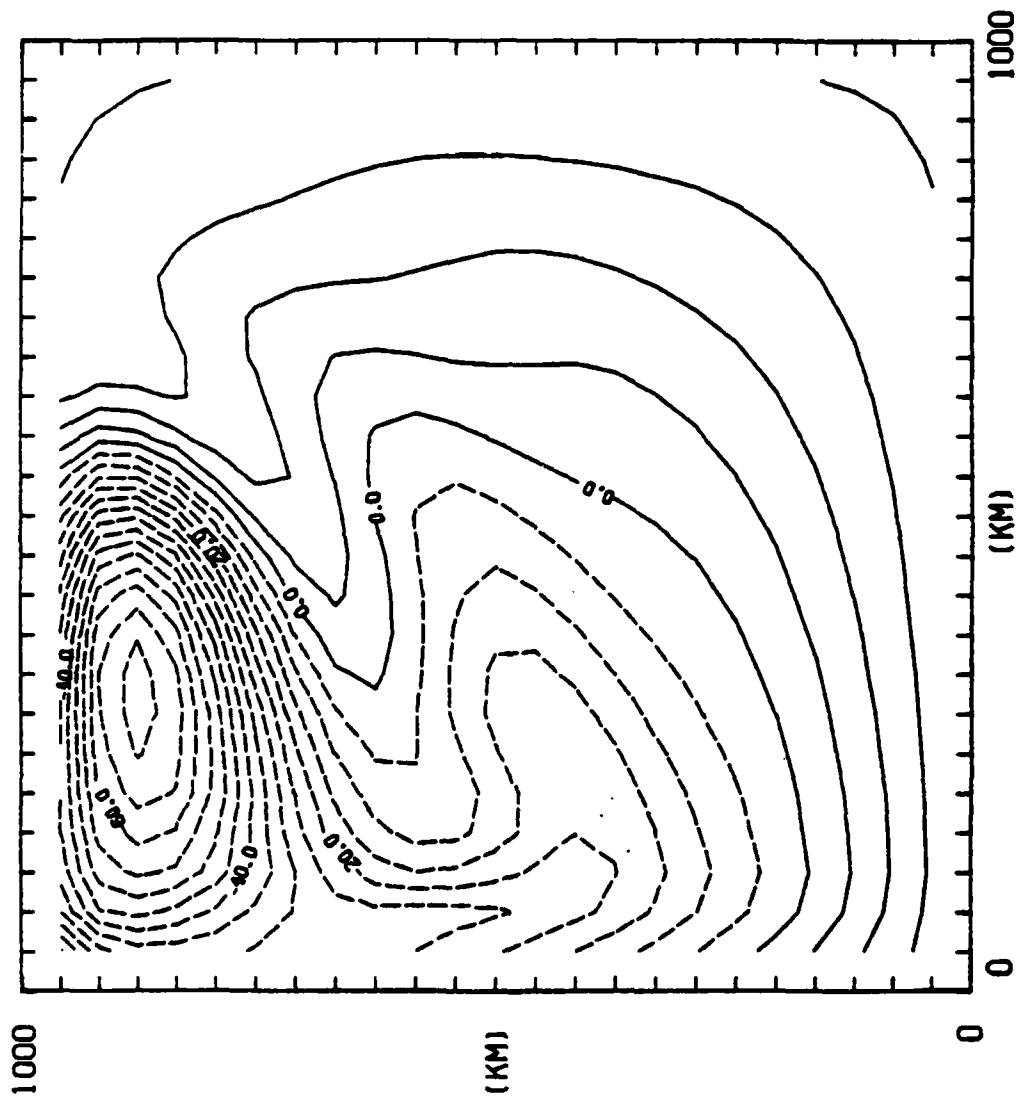
CONTOUR FROM 0.0000 TO 2.0000E-01 WITH AN INTERVAL OF 0.1000E-01

# UPPER LAYER MEANFLOW S. GYRE HOLLAND



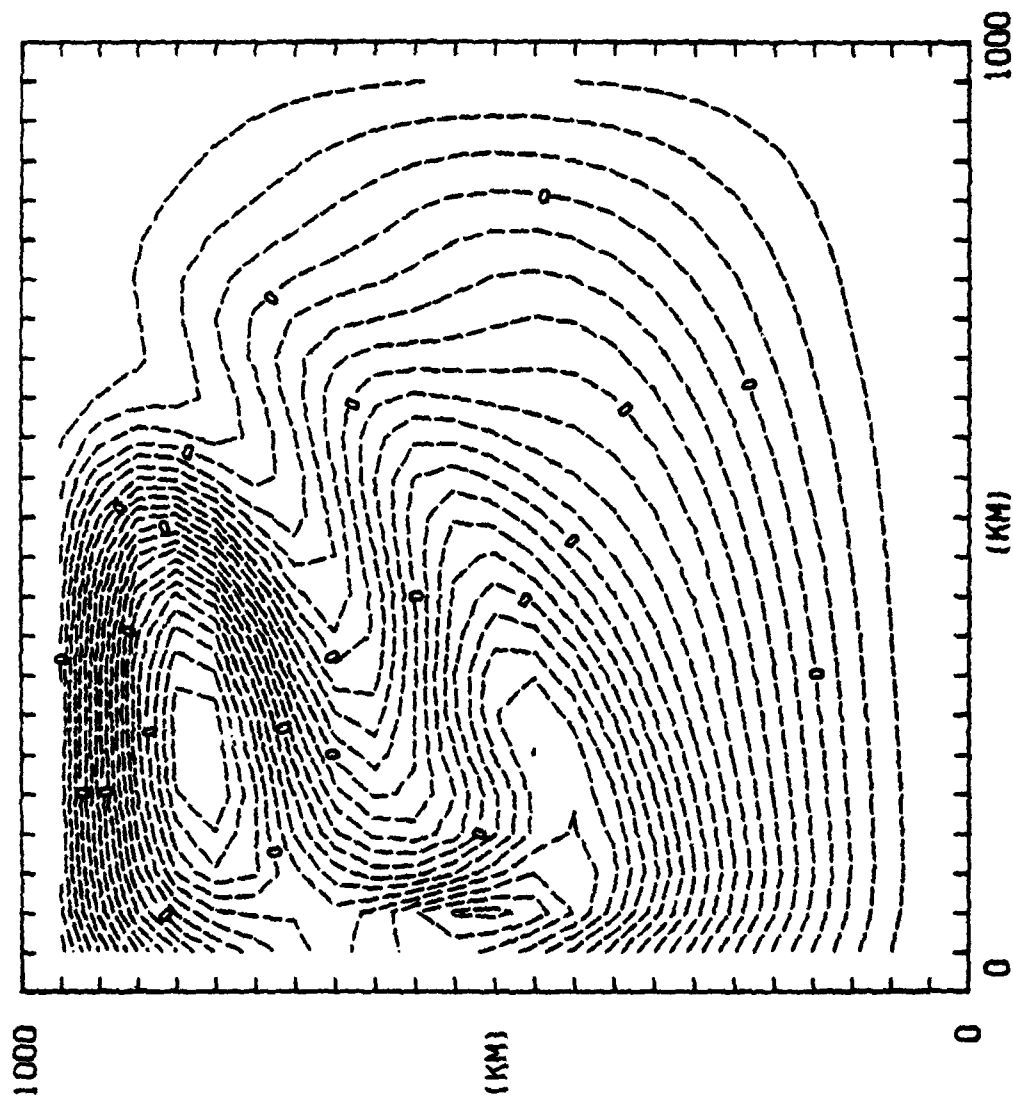
CONTOUR MAP -12.000 TO 24.000 CONTOUR INTERVAL OF 6.0000 PT(15.31) -0.0002

# LØWER LAYER MEANFLØW S. GYRE HØLLAND

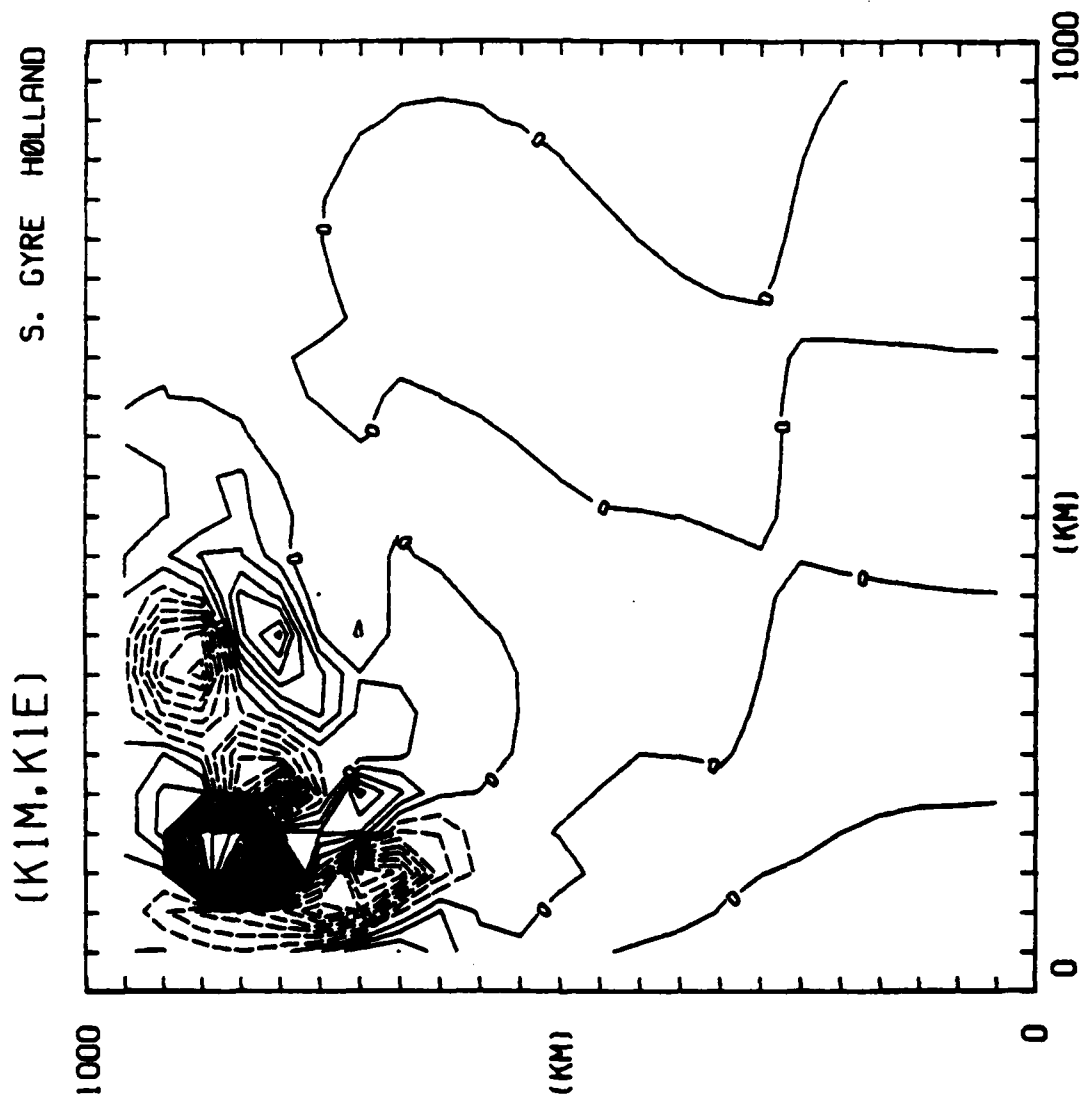


CONTOUR FROM -20.000 TO 20.000 CONTOUR INTERVAL OF 5.0000 P1(2.31) 11.000

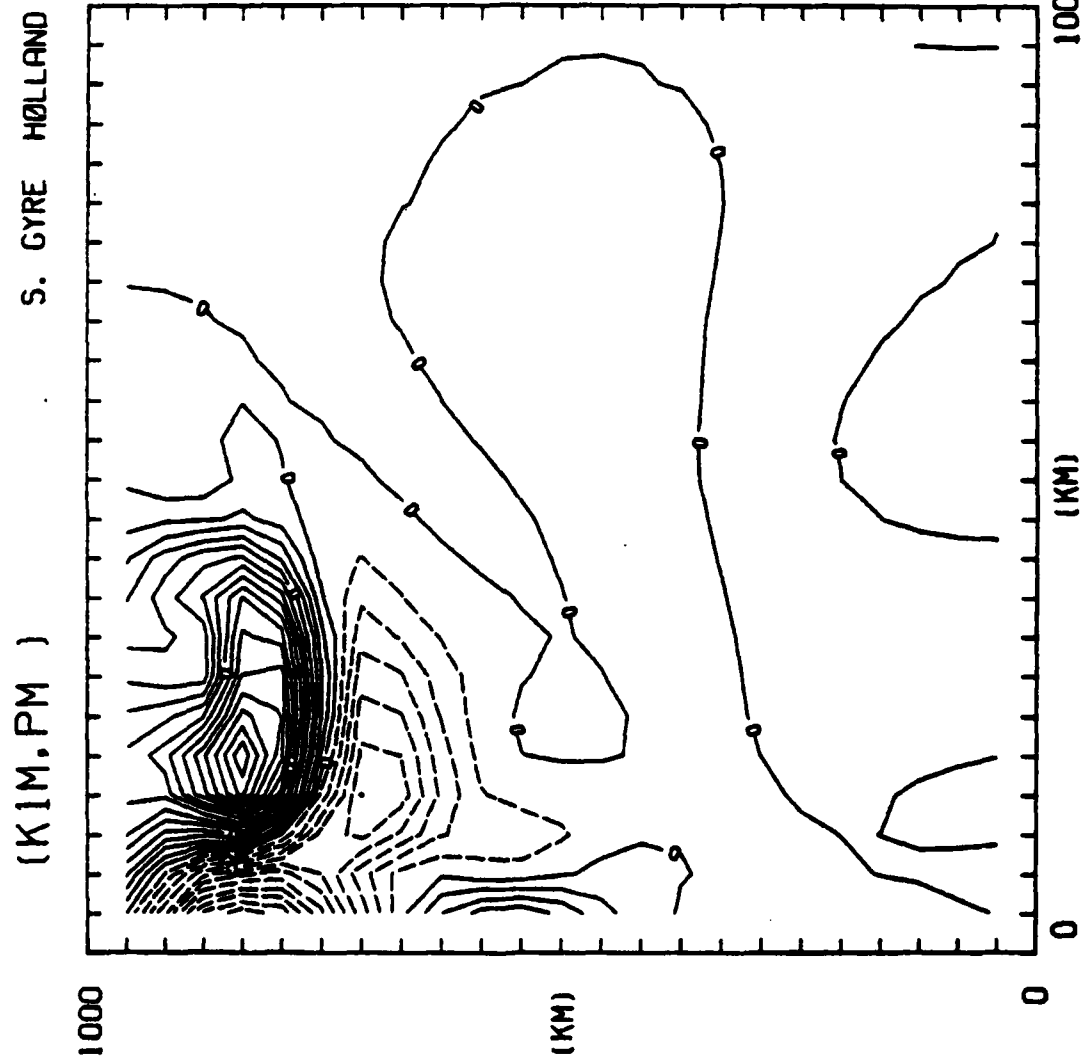
(K1M.S WIND ) S. GYRE HOLLAND



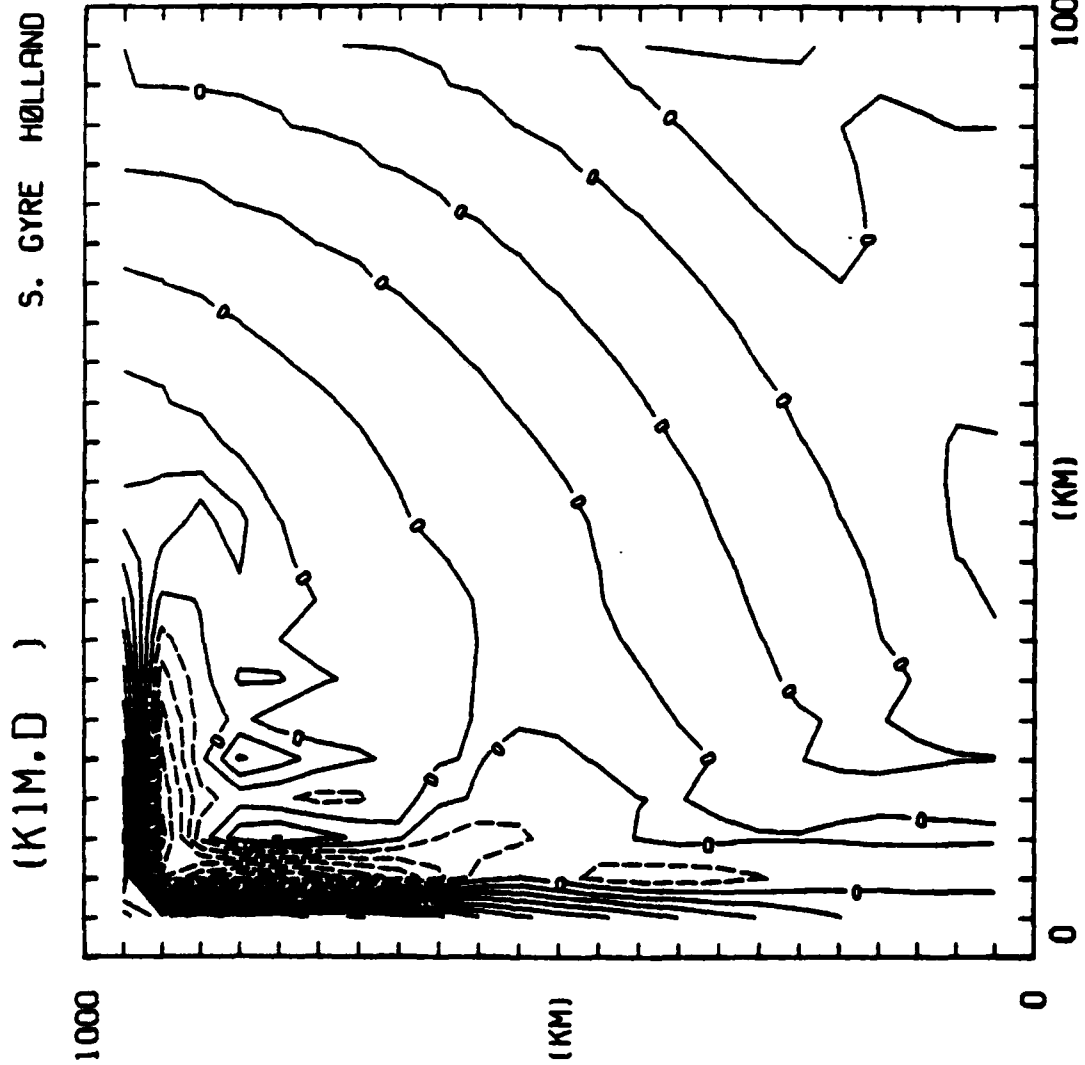
CONTOUR FROM -0.00000E+00 TO 0.00000E+00 WITH 0.00000E+00 SPACING ON LEVELS SET BY 0.10000E+00



CONTOUR FROM -0.11800E-04 TO 0.10000E-04 CONTOUR INTERVAL OF 0.10000E-05 PT(12,31) = -0.17753E-07 UNITS 3 SCALED BY 0.10000E-08



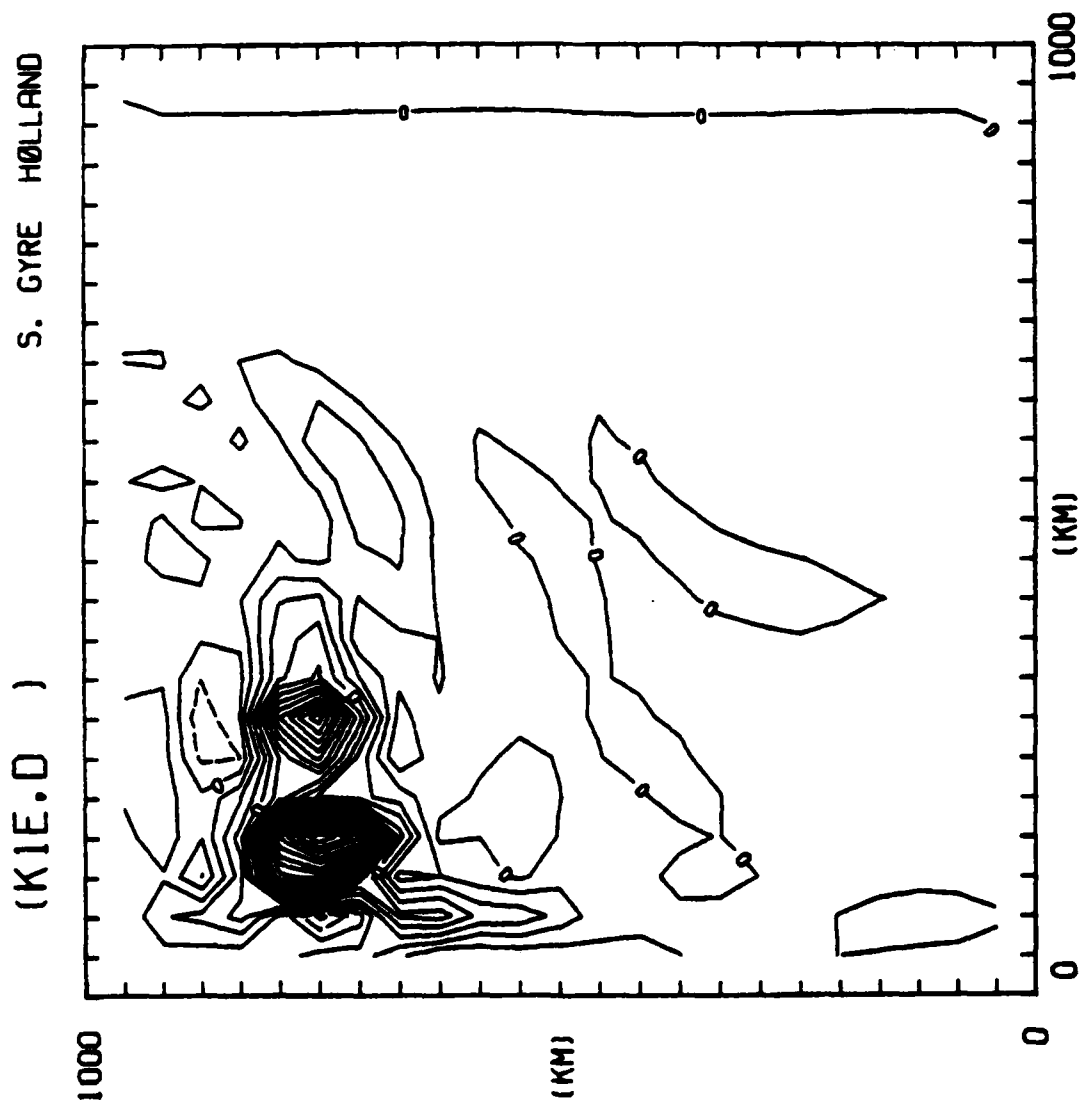
CONTOUR FROM -0.11000E-04 TO 0.11000E-04 CONTOUR INTERVAL OF 0.00000E-05 P113.D1= 0.00000E-05 LEVELS SCALED BY 0.10000E-05



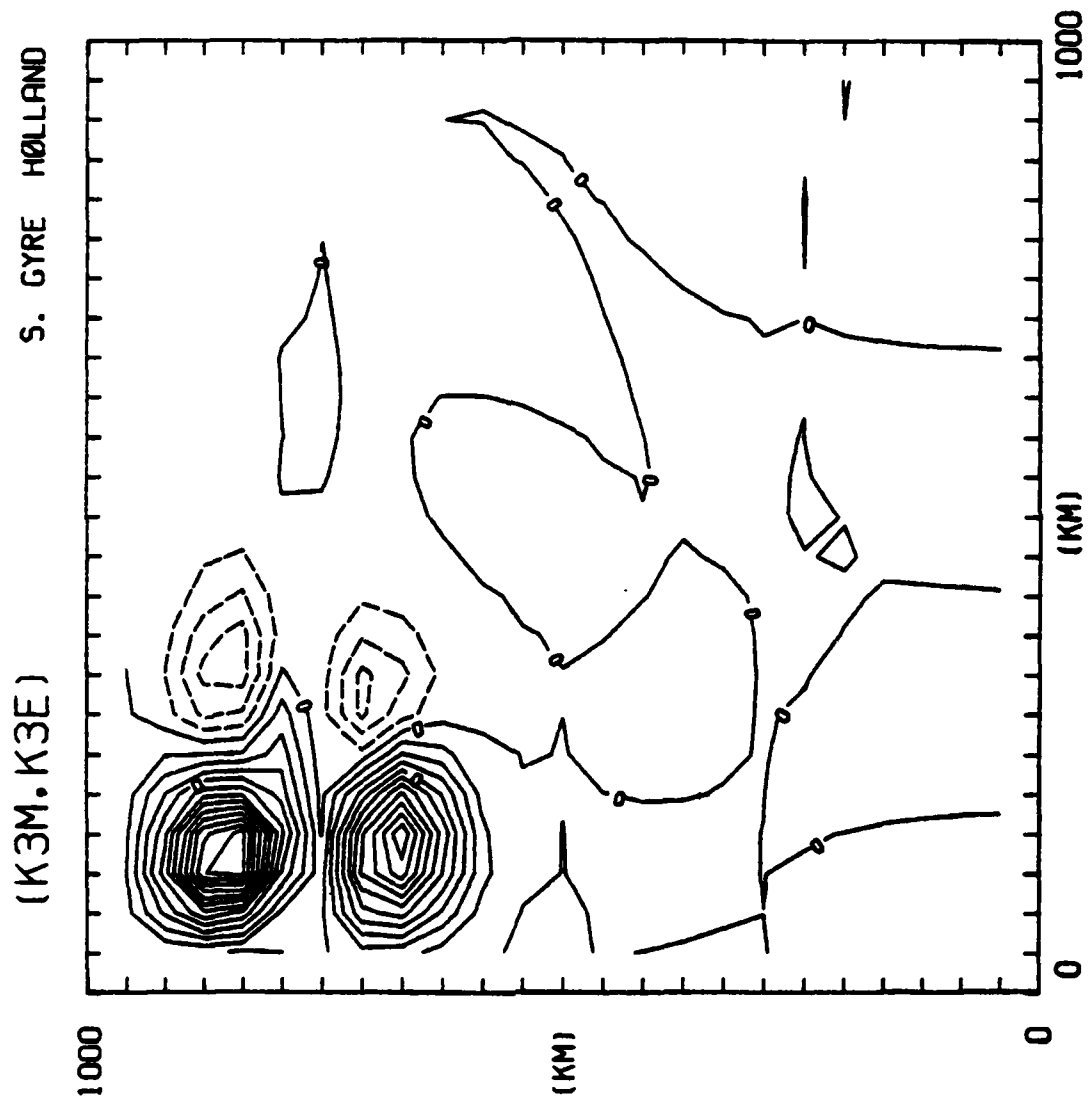
CONTOUR FROM -0.00000E+00 TO 0.00000E+00 contour interval of 0.00000E+00 P1(0.0) = -0.00000E+00 LEVELS SCALED BY 0.00000E+00



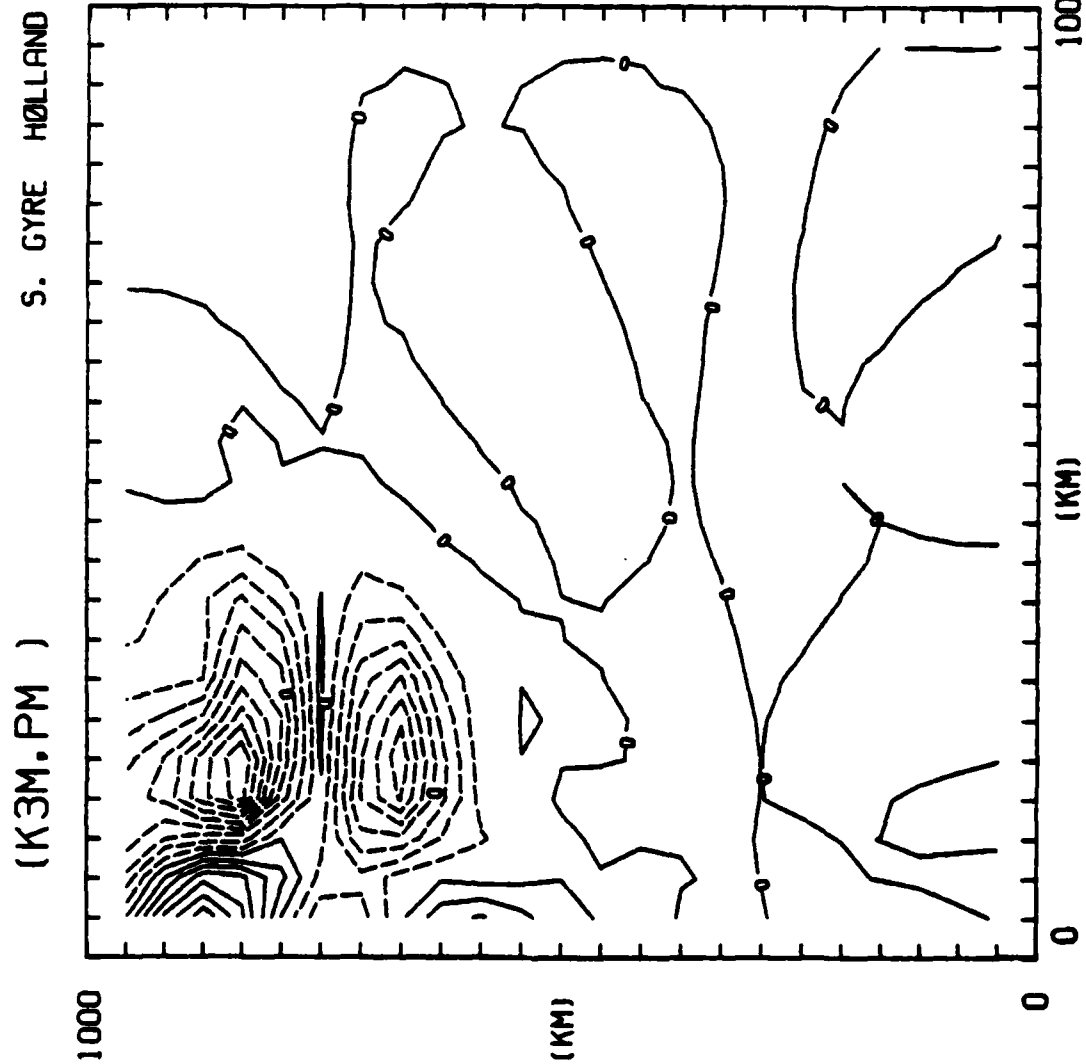




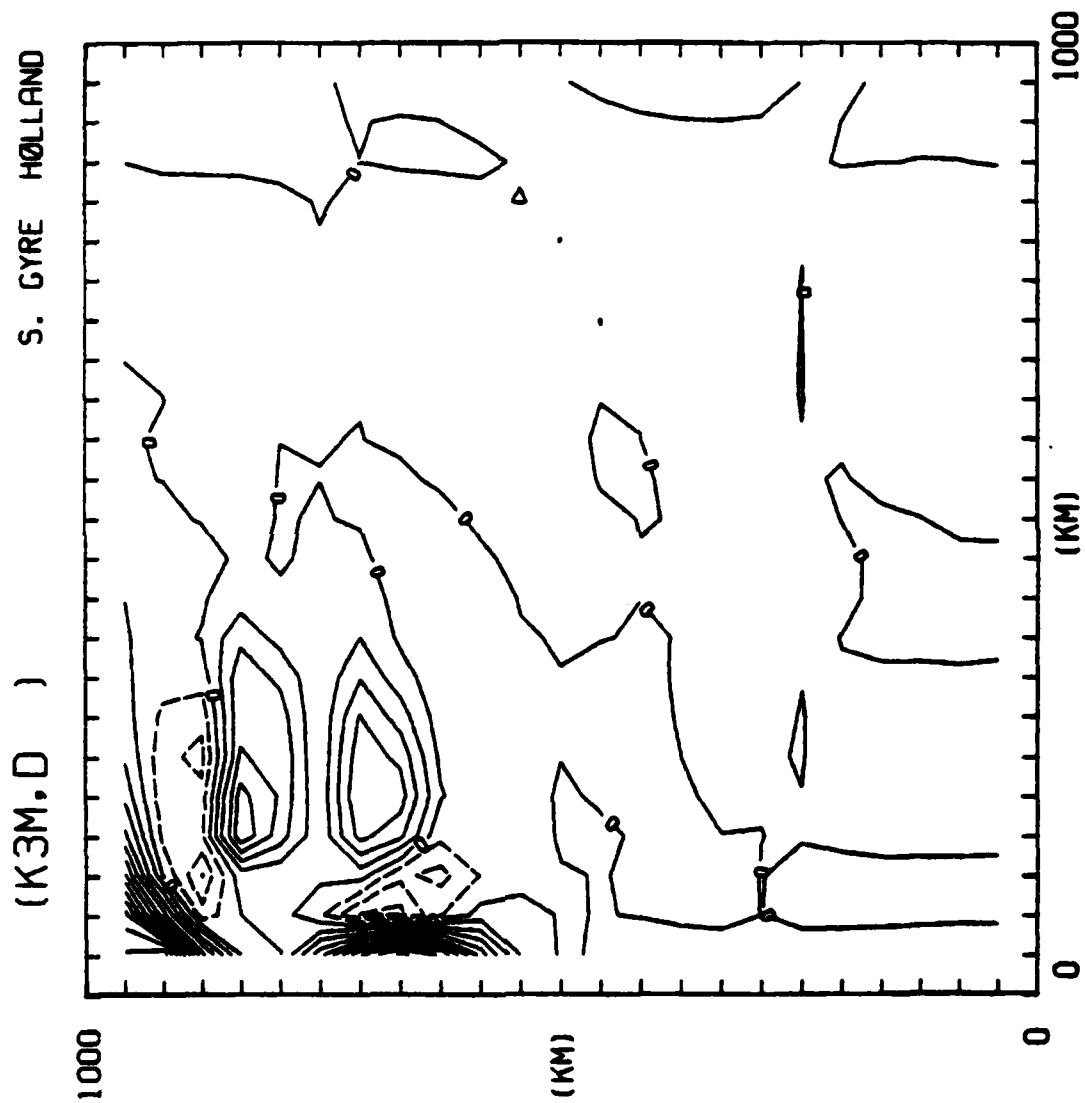
CONTOUR PLOT -0.4000E+00 TO 0.3000E+00 CONTOUR INTERVAL OF 0.2000E+00 P112.D1= -0.3000E+00 UNITS SCALED BY 0.1000E+00



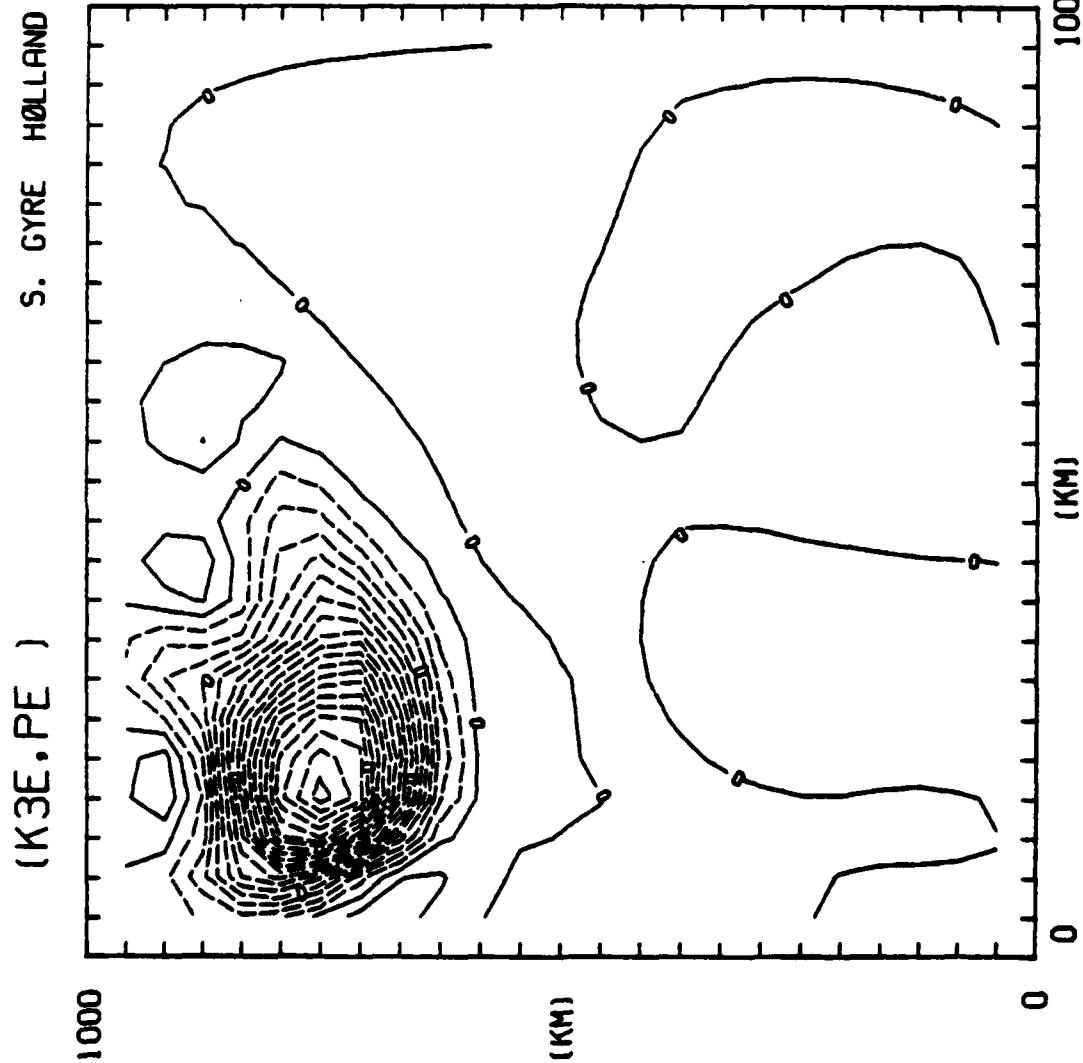
CONTOUR FROM -0.10000E-08 TO 0.70000E-08 CONTOUR INTERVAL OF 0.00000E-07 P(15,3)= 0.31700E-08 UNITS SCALED BY 0.10000E 10



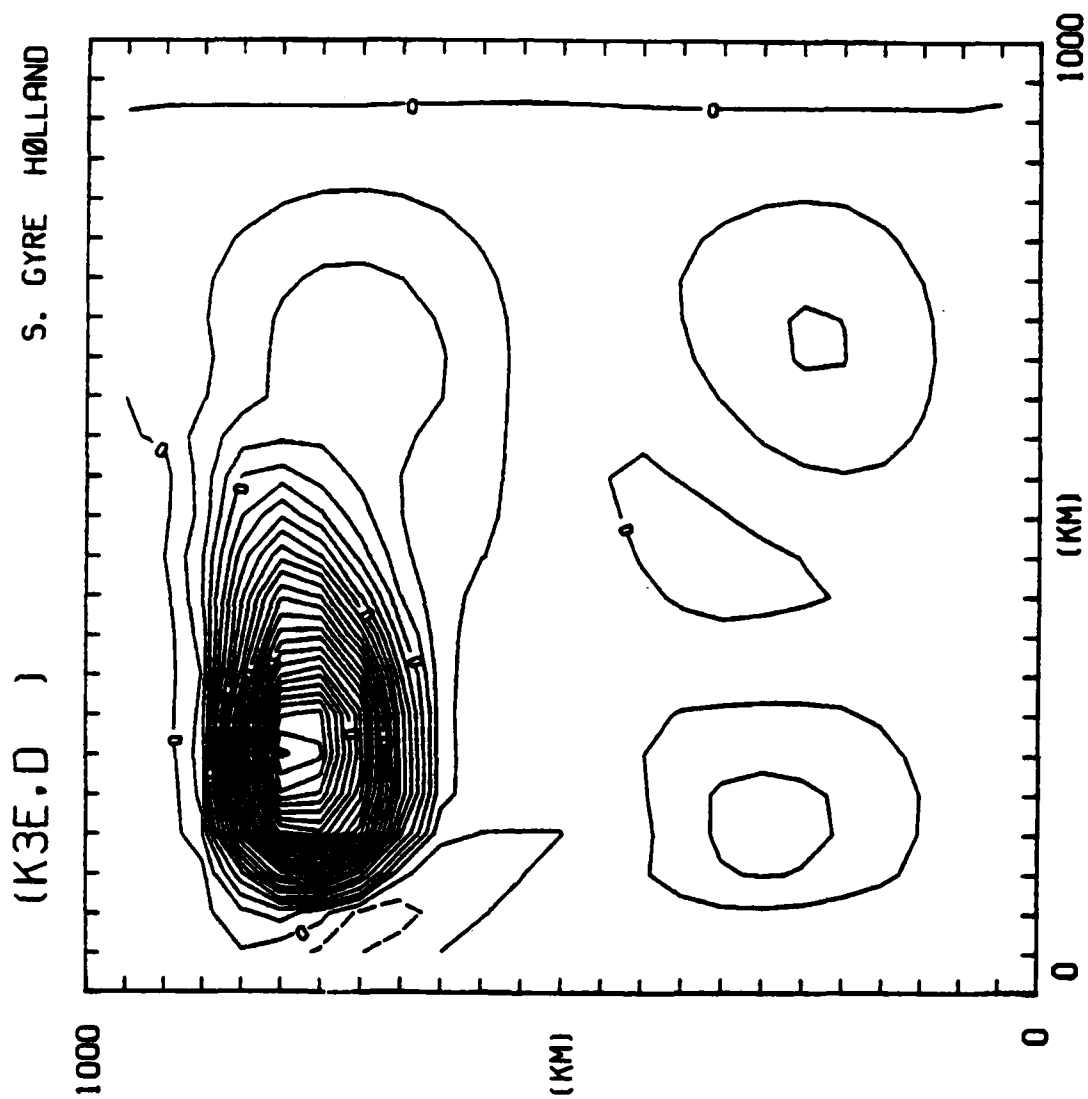
CONTOUR FROM -0.00000E+00 TO 0.00000E+00 CONTOUR INTERVAL OF 0.00000E+00 P113-310 0.00000E+00 UNITS,3 SCALES BY 0.00000E+00



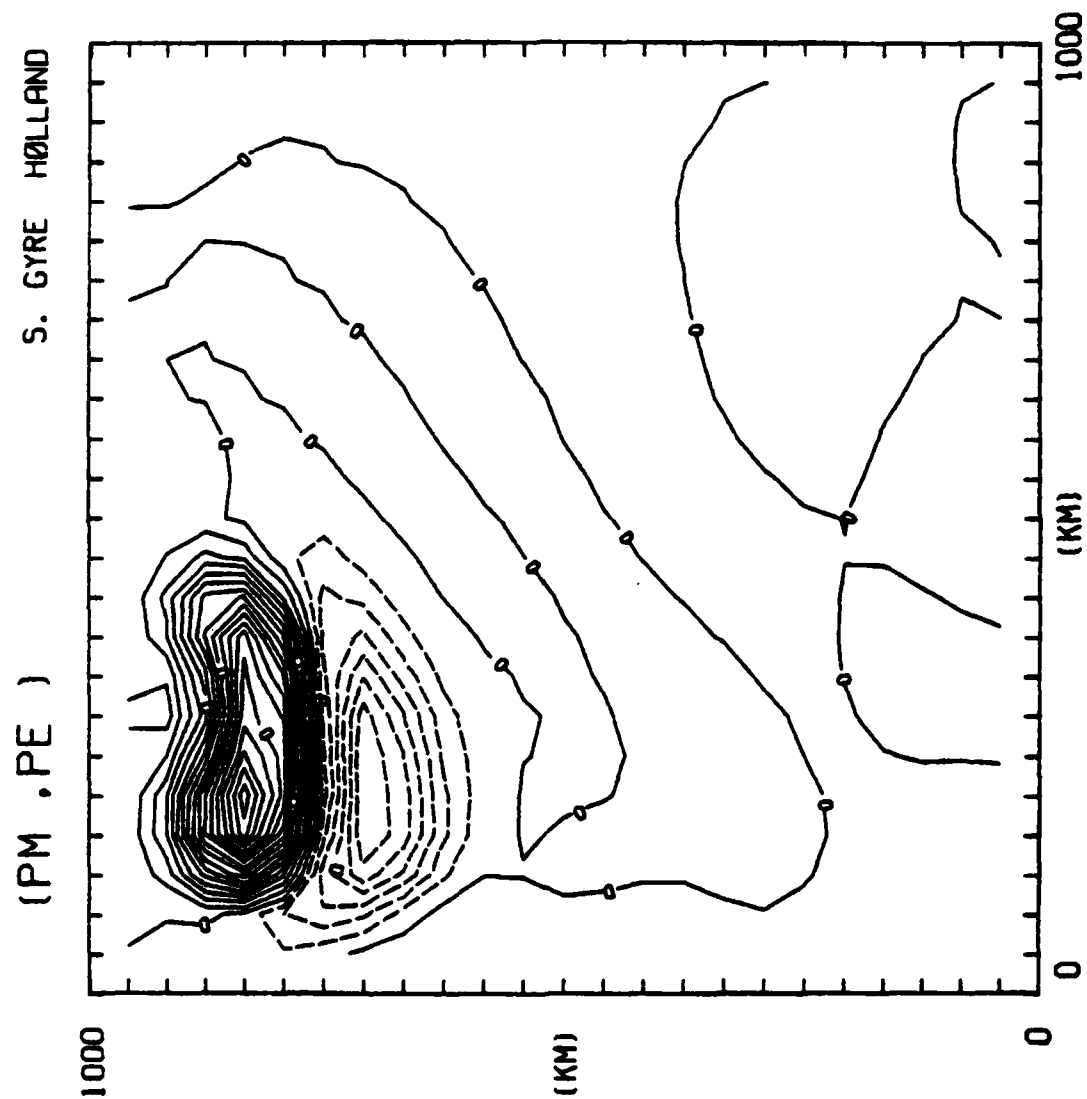
CONTOUR FROM -0.20000E+00 TO 0.00000E+00 CONTOUR INTERVAL OF 0.00000E+00 PT(10,10) = -0.14444E+00 UNITS SCALED BY 0.10000E+00



CONTOUR PLOT -0.3000E-06 TO 0.4000E-06 CONTOUR INTERVAL OF 0.2000E-06 P112.31= 0.3000E-06 LANDS SCALED BY 0.1000E 00



CONTOUR FROM -2.10000E-08 TO 0.20000E-05 contours interval of 0.10000E-08 P(12,3)= 0.11010E-07 UNITS SCALED BY 0.10000E-08



CONTOUR FROM -0.0000000000000000 TO 0.0000000000000000 WITH 0.0000000000000000 SCALED BY 0.0000000000000000

**END**

**FILMED**

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**DTIC**